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U. S. DEPARTMENT OF AGRICULTURE,
WEATHER BUREAU.

STUDIES ON THE STATICS AND KINEMATICS OF THE ATMOSPHERE IN THE UNITED STATES.

Reprints from the Monthly Weather Review, January to July, 1902.

BY

FRANK H. BIGELOW, M. A., L. H. D.,
PROFESSOR OF METEOROLOGY.

PREPARED UNDER THE DIRECTION OF
WILLIS L. MOORE,
CHIEF U. S. WEATHER BUREAU.

WASHINGTON:
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STUDIES ON THE STATICS AND KINEMATICS OF THE ATMOSPHERE IN THE UNITED STATES.¹

I.—A NEW BAROMETRIC SYSTEM FOR THE UNITED STATES, CANADA, AND THE WEST INDIES.

PRELIMINARY REMARKS.

On January 1, 1902, at the 8 a. m. observation, seventy-fifth meridian time, a new system for the reduction of the station barometric pressures to the sea-level plane, was put in operation for the United States, Canada, and the West Indies. The daily weather maps used in forecasting the intensity and the path of storms, and the other allied phenomena, are therefore constructed upon a basis differing from any hitherto used. Students who consult the published weather maps should remember that the series terminating with the above date is not comparable with the others following it, the difference at some stations on the Rocky Mountain Plateau for certain seasons of the year amounting to several tenths of an inch of pressure by the mercurial barometer. The problem of reducing the pressures observed at stations located on the Rocky Mountain Plateau to sea level has always been recognized as one of unusual scientific difficulty, and it has been under discussion in the Washington Office at intervals ever since the establishment of the Government service. So far as can be judged at the present writing the success of the new system is assured, and if this favorable opinion is confirmed by continued use, it will mark the termination of thirty years' effort to solve this question in a practical form. The other plateau districts of the world, Mexico, South America, especially Argentina, south Africa, Australia, and southern Asia, will doubtless profit by the experience of the United States Weather Bureau, on consulting the solution adopted for the United States, Canada, and the West Indies.

Prof. R. F. Stupart, Director of the Canadian Meteorological Office, has courteously cooperated by supplying the necessary data for the Canadian stations, since the common interests of both countries require the adoption of the same methods of barometric reductions. There is no task properly belonging to the Weather Bureau upon which more time and labor has been expended than upon this problem, and the present discussion is the sixth well defined attempt to reach a satisfactory conclusion. The importance of putting the barometric pressures on the elevated plateau, covering one third of the territory for which the official forecasts are made, on a satisfactory scientific basis, fully justifies this work, because it is of primary importance not to attribute to weather conditions any pressure changes that are in reality due to the method of reduction to the plane of reference.

The eastern and central portions of the United States and Canada are generally at levels less than 1,000 feet above the sea, and also the Pacific coast is at low level, so that for these districts the barometric reduction offers no difficulty. Between these, throughout the Rocky Mountain region, there is a rough country where the stations are at different elevations up to 7,000 feet, where the surface temperature conditions range enormously, say from -40° F. to $+60^{\circ}$ F. on a single map in

extreme cases, where the prevailing winds from the Pacific Ocean produce one type of weather on the western slopes of the mountains and another on the eastern, to say nothing of the effect of great arid districts between them, and where the configuration of the mountain valleys, in which many of the stations are located, relative to the neighboring ranges rising up to 12,000 or 14,000 feet in some cases, causes various local peculiarities in the behavior of the barometer.

A description of the construction of our new station pressure normals is properly a preliminary to the solution of the plateau problem. In the years between 1871–1880, while the barometric network was being extended over the plateau districts, many of the elevated stations were at the Army posts where no measurement of the altitude had been made, except by the barometer. We now know that several of these early elevations were seriously in error, say from 10 feet up to 200 feet, and as a change of 10 feet in altitude corresponds approximately to 0.010 inch pressure, the irregularities on the sea-level plane arising from this source alone were not inconsiderable. The gradual extension of the various surveys by the Government over the plateau, together with the railroad levels executed and revised by the different companies, have gradually built up a system of check levels at intersecting points, with accurate differential levels between them, so that now the absolute elevations of the several stations have been determined with much accuracy. An adjustment of these levels was made by Prof. Cleveland Abbe in 1871–72; the work was then taken up by the Geological Survey, and the latest results of these surveys are given in Gannett's Dictionary of Altitudes, edition of 1900. The Weather Bureau was supplied with the corrected altitudes before the publication of this report by the Geological Survey, so that we have had the advantage of this data from an early stage in our own work.

THE ADOPTED STANDARD ELEVATION FOR THE EPOCH, JANUARY 1, 1900.

Besides the incorrect actual elevations which, for one reason or another, have been adopted during thirty years, there have been numerous changes in the elevation of the local offices of the service at the same station, involving many small variations in the altitude above sea level. A very careful reexamination of the station records of the respective stations showed that it was practically impossible to assign correct absolute elevations for the several changes as referred to the sea level, but that it was possible to discover the differences by which the successive changes in height followed each other (that is, the height of the barometer in the new office above or below that in the old office), the series of variations giving a chain of steps up and down in the succession of changes. These were carefully determined, and they were then applied to the elevation occupied by the station at the epoch January 1, 1900, so that the actual heights were thus found for the respective intervals during which the barometer remained in one position, and

¹ Reprinted from the Monthly Weather Review for January, 1902.

they were referred in this way to our latest and best elevations as given by recent surveys. Having adopted the elevation for the station at the given epoch, all the recorded actual pressures were reduced to the elevation of 1900 by small differential pressure corrections, so that the entire pressure system becomes homogeneous for the station.

During the years following 1900 a similar plan is to be followed, and all pressures will be reduced back to the standard elevation, so that the series will be maintained strictly comparable throughout the life of the station itself. There is great advantage in this procedure, for two reasons. It was found that in the other attempts to construct pressure normals the earlier computations were readjusted to the latest elevations at the different dates, thus obscuring the record and consuming a great amount of labor without arriving at final results. Also, the reduction tables to sea level, provided for the use of the stations, had to be renewed with every removal, which also consumed much time. On the new plan, however, each year's observations is added directly to a homogeneous station system, and the same reduction table serves without modification in consequence of any local changes. Indeed it is absolutely essential to reach such a basis of operation in meteorology as this, if there is to be made possible a scientific study of the secular variations of the weather, that is, the large problem of why and how the seasons, the climate, and the crops, differ from year to year, this being the next great problem awaiting practical meteorology. Evidently all the cosmical questions involving variations in the radiations of the sun must be compared with as definite a pressure system as this, if scientific results are to be secured from the meteorological data. It may be stated in passing, that in recent years, since the Government has erected large buildings in the cities of the United States, the Weather Bureau offices have been more permanently located, and that the average series of unbroken observations is growing longer than it used to be 10 and 20 years ago. At the same time the elevations are a little higher, because the offices are usually placed in the upper rooms of the lofty federal buildings.

OTHER CORRECTIONS TO THE STATION PRESSURES.

Besides reducing the observed pressures to an adopted station elevation it was necessary to make several more corrections in order to obtain a homogeneous system of normals. (1) The records were thoroughly inspected for the several corrections which ought to be applied to the barometer readings, and we have now a complete list of the barometer numbers and their errors for capillarity, scales, etc. Besides eliminating a few mistakes, there were two important special corrections to be applied. During the interval 1873-1878 a correction of 0.013 inch had been added to the Signal Service standard barometer to reduce it to the supposed Kew standard, but a system of comparisons instituted in 1877-78 showed that this was probably an error, and I have, therefore, removed it from the new series. A policy prevailed in the office from 1888 to 1898 to the effect that small errors could properly be neglected in the barometer reductions, and in accordance with it all corrections for scale error and capillarity smaller than ± 0.007 inch were discarded; these have now been all restored. (2) The correction to standard gravity, at sea level on the forty-fifth parallel of latitude, was applied during some years and omitted during others, so that there was irregularity in this respect. The gravity correction has now been systematically added by me since the beginning of 1873. (3) The hours of simultaneous observation have been changed several times since the opening of the service, but practically the observations can be grouped in two series of selected hours, 7 a. m., 3 p. m., 11 p. m., till June 30, 1888, and 8 a. m., 8 p. m. since that date. Referred to the mean of 24 hourly observations which is the natural standard to adopt for the world, these two systems present very different types of corrections for the North American Conti-

nent, and they must be reduced to some one system in order to be comparable. Accordingly auxiliary tables were prepared by which observations at a few selected hours could be reduced to the mean of 24 hourly observations, and the different series have been so corrected since January 1, 1873. These 24 hourly corrections will be applied in the future to all monthly and annual pressures published by the Weather Bureau, so that the fundamental system may remain intact in case other hours of observation should ever be adopted, differing from those now in use.

The application of the corrections for local elevation, scale error, capillarity, instrumental temperature, gravity, and diurnal variation, to the barometric readings, gives a smooth homogeneous system of values, from which the mean annual and the mean monthly station normals were derived and checked by cross addition; they are noted as *B*. From these the annual and the monthly variations from the general mean were obtained, and they have been thoroughly discussed in the Report of the Chief of Weather Bureau, 1900-1901, Vol. II. In order to determine our final station normals, *B_n*, it was further necessary to reduce all the short series to a standard fixed by the long 27-year series for a large number of stations sufficient to control the work. There are about two hundred and sixty-five stations, including the Canadians, to be dealt with, and of these about seventy-five had a long record of twenty-seven years. The run of the monthly residuals increases in irregularity as the number of years of observation decreases, but we have so managed the discussion that a short series normal can be reduced to the long series normal, and thus the station placed upon the standard basis. Whenever a new station is opened by the Weather Bureau a standard normal pressure can now be constructed by a brief computation, and the normal is more accurate than any that could be obtained by fifteen years direct observations, since these take up all the turbulent pressure fluctuations due to the general and local circulations, which it is impossible to eliminate, except by the use of the observations of many years. I may add that my experience with the barometric observations of the United States convinces me that they have always been of a high order of scientific excellence, and that the apparent residuals are not in fact due to accidental irregularities, but possess general and even cosmical significance when they are thoroughly discussed. It has been a mistake to assume that they are not worth the most exact treatment in the reductions; on the other hand there is every reason to believe that they will become of prime importance in the solution of several solar-terrestrial problems.

THE SEA LEVEL, 3,500-FOOT AND THE 10,000-FOOT PLANES OF REFERENCE.

Having obtained these reliable station pressures throughout the United States and Canada, the plateau problem now comes before us for discussion, in order to reduce the pressures taken at different elevations to the adopted planes of reference, in our case to the sea-level plane, to the 3,500-foot plane and the 10,000-foot plane. All the forecasting problems have been heretofore studied solely on the sea-level plane. But it seems evident that our grasp upon the weather problem will be greatly strengthened if we can study at least three sections through the atmosphere daily instead of the one at the bottom of it. I selected the 3,500-foot plane because this is the average height of the Rocky Mountain stations, to which the least possible reduction is required; also, because it is the average altitude of the base of the cumulus cloud sheet over the eastern districts, upon which observations can be most favorably made with theodolites for gradients of pressure, temperature, and vapor tension. Besides this, it is the altitude at which the moving currents of air are sufficiently distant from the ground to take on their natural configuration when freed from the surface turbulent friction. The 10,000-foot plane was chosen because it is already in use by the MONTHLY WEATHER REVIEW

to show the monthly mean isobars at a considerable altitude. Furthermore, it is just in the midst of the most rapid-moving horizontal local currents, which build up the cyclones and anticyclones of the middle latitudes and upon which the intensity of storms depends. We know that the isobars on these three planes differ considerably from one another, the closed curves on the lower plane tending to open out into long sweeps on the upper plane, and it is probable that an intercomparison of these varying isobars from map to map will be valuable.

THE NEW REDUCTION PRESSURE TABLES.

It is evidently necessary to possess reduction tables of a perfectly general and flexible kind in order to make the necessary reductions from the several stations to these three planes, and from one plane to the other, in either direction upward or downward. As there are no such tables in print, I have first computed logarithmic reduction tables in English measures, similar to those in metric measures, described in the International Cloud Report of 1898-99, the intervals being for every 100 feet up to 10,000, and for every 10° F. from -40° to $+100^{\circ}$. From these general tables the special station tables were made, giving the corrections to be applied to the station pressure at intervals of 0.20 inch to reduce it to the three planes, respectively. These individual tables contain a correction for the humidity term separated from the dry-air term, a correction for the plateau effect, a residual reduction for a few stations, and two temperature arguments—first, the mean temperature of the air column, and second, the corresponding surface temperature, which is the mean value of two successive 8 o'clock observations, the last always including that hour for which the reduction is made. In order to simplify matters as much as possible for the observers on the stations, the individual station tables are constructed by combining all these corrections and applying them at short intervals of the station pressure, namely, for every tenth of an inch, and for such close intervals of the temperature argument that there shall be no interpolation necessary in this direction in order to obtain the hundredth of an inch of reduced pressure. The result is contained in three tables, one for each plane, with the surface temperature and station pressure as the arguments, and the reduced pressure to the three planes, respectively, in the body of the table, instead of the correction to the observed pressure. There is thus no computation to be done at the station to reduce the observation, and the time consumed in preparing this portion of the cipher code message is very short. The special tables for the stations for use in reduction to the 3,500-foot and the 10,000-foot planes are now being made up, the first and the second forms leading up to them being completed and checked. The tables for reduction to sea level are already in operation, and, so far as known, there is no occasion to modify the reductions at any of the stations. When one considers the large amount of painstaking and careful labor required to produce such a result as this in so complex a problem, it is a pleasure to commend the faithful work of Mr. Heiskell and Miss Hawkins, who have been my assistants in this computation. We hope to be able to make a trial of the working of the pressures on the higher planes before very long.

PREVIOUS DISCUSSIONS OF THE PLATEAU PROBLEM.

After all these preliminary matters have been concluded we may proceed to the really difficult portions of the work. They group themselves around three points, (1) the proper relation between the observed surface temperature, t , and the mean temperature of the air column, θ , corresponding to and substituted for the plateau at the regular intervals for which the general logarithmic reductions were computed; (2) the effect of the plateau itself upon the free air pressure; (3) the residual local effects which can not be classified with the other reductions. These will become clearer to the reader by briefly mentioning

the previous methods which have been employed in reducing the plateau pressures to the sea-level plane. (1) From 1871 to June, 1881, the old Guyot tables were used in reducing low-level stations, with the surface temperature and pressure at the time of observation as the argument. Certain annual constants were employed in the cases of high stations. The effect was to cause the isobars to swing widely between the morning and evening hours, and generally the maps were very unsteady. (2) July, 1881, to June, 1886, monthly constants were used for each station, as recommended by the first board on barometer reductions; a single constant answered for each month; these are sometimes known as the Abbe-Upton constants. (3) July, 1886, to June, 1887, the entirely new system of tables by Professor Ferrel was used, thus introducing several valuable principles. Thus, the mean temperature of the preceding twenty-four hours was used instead of that belonging to the respective hours of observation; this was reduced by a vertical temperature gradient, 0.165 per 100 feet, to the approximate mean of the column; the pressure and temperature arguments (B , t) were both employed in entering the table; a special correction for the plateau effect was made in the form $C \cdot \Delta \theta \cdot H$, where $C=0.00105$, $\Delta \theta$ is the variation of the temperature from the annual mean, and H is the altitude in units of a thousand feet. The application of the correction for the plateau effect removes the wide range in pressure which occurs on the plateau between summer and winter and reduces it to about the same value on the plateau and in the low level eastern districts. For example, if the mean annual temperature is 50° , that for January 25° , and for July 80° , at a station 5,000 feet above the sea level, we have $0.00105 \times (-25) \times 5 = -0.131$ inch for January, and $0.00105 \times (+30) \times 5 = +0.158$ inch for July. The annual range for high stations on the plateau is about 0.400 inch, and on the low levels it is only 0.150, the difference being simply the plateau effect. Professor Ferrel's tables were not used very long. (4) July, 1887, to December, 1890, a mixture of Ferrel's and Hazen's tables; 1891-1901, Hazen's tables. Professor Hazen constructed a general empirical formula with the object of simplifying the form of the station table. For this purpose he assumed that Mount Washington is the type for the plateau reductions, which is in fact erroneous, since that isolated mountain acts like a free air point, except for the modified value of θ ; he assumed that the sea-level pressure should always be exactly 30.00 inches, and at the same time abandoned the pressure argument entirely, with all depending upon it, and computed the correction under these conditions; he rejected the plateau effect correction, and at the same time the change of surface temperature to the mean temperature, θ , was neglected. On applying this system to the daily map it was necessary to make certain arbitrary changes in the computed reductions in order to produce smooth isobars. The great simplicity in the use of this table, having only the surface temperature as argument, seems to have been considered sufficient ground for substituting these tables for Ferrel's, so that from 1891 till 1901, inclusive, they have been employed in making the daily weather maps, although well known to be unscientific and inaccurate. However, it should be said that although the plateau correction was omitted, the practical working of the Hazen method was such as to make the sea-level reductions conform much more closely to the Ferrel system than to the pure Laplacean system, which is correct for free air reductions only. On this account the Ferrel and the Hazen systems work in the same direction for wide departures of the temperature from the annual mean, and to some extent relieve the plateau exaggeration, so that we conclude that the weather maps have served fairly well for the practical purposes of forecasting. (5) 1895-1896, Professor Morrill, in connection with a second board on barometry, rediscussed the problem and computed a set of tables which have not been published, though they have been used for some office work, especially the construction of the sea level and the 10,000-foot

plane maps for the MONTHLY WEATHER REVIEW during 1896-1901. The Laplacean free air reduction was computed by special tables for the pressure and the temperature arguments, the value of θ being found by certain adopted average vertical gradients varying for the different seasons of the year; the humidity term was made so as to modify the logarithmic argument; the plateau term was entirely omitted; the tables were in the form of a logarithmic argument, which was not very convenient for rapid work. It was suggested at the same time that a system of constants, daily rather than monthly, be resumed for making the necessary forecasting isobars.

BIGELOW'S SYSTEM OF BAROMETRY, 1902.

We now come to the sixth attack upon the problem, and shall here merely enumerate the steps in the discussion, while the report itself will be found in Volume II, Annual Report, Chief of Weather Bureau, 1900-1901. In substance the principles laid down by Ferrel have been adopted, but the work has been carried far beyond the degree of perfection possible to him nearly twenty years ago, in consequence of the numerous observations at our disposal, whereas Professor Ferrel contented himself with only four years of observation at the plateau stations preceding the time of his studies.

THE SEA-LEVEL TEMPERATURES.

The object to be obtained is to separate the temperature argument from the plateau effect, and to arrive at smooth isobars in correct relations to the winds and the weather throughout the Rocky Mountain region. Having prepared the monthly station pressure normals, as described above, the corresponding station temperature and vapor tension normals were extracted from the office records. The plateau is therefore to be considered as dotted over with 60 or 70 stations where the monthly values of the elements (B , t , e) are known. Assuming an average vertical temperature gradient of 0.30° per 100 feet, the temperatures were first reduced from those given at the station elevation, to corresponding values at selected heights, 500, 1,500, 6,500 feet, through short distances; for example, all between 0 and 1,000 feet were corrected to 500 feet, and so on. This concentrates the reductions on a few planes. Then a preliminary set of temperature gradients in latitude and longitude was computed from the temperatures on these few planes, throughout the region west of the Mississippi Valley. Certain *centers of reduction* were taken, namely, where the one hundred and twentieth meridian crosses the fiftieth, forty-fifth, fortieth, and thirty-fifth parallels of latitude, and the one hundred and tenth, one hundredth, and ninetieth meridians cross the same parallels, and the temperatures were reduced by the two horizontal gradients to these centers, so that a series of temperatures varying with the altitude are now known in vertical directions, at about 18 geographical points. These temperatures were plotted on a diagram whose abscissas are temperature values and whose ordinates are altitudes, one chart for each month and one for the year; average curves were drawn through the plotted temperatures and prolonged by best judgment to the sea level. In the majority of cases it was easy to do this, as the curvature was distinctly developed on the diagrams. In this way sea-level temperatures were found at several evenly distributed points beneath the plateau, and they were transferred to monthly charts on the sea-level plane, which were completed for the Pacific low level districts and for the central and eastern portions of the United States and Canada. A system of well graded isotherms was drawn through them for the entire country. Small adjustments of the temperatures on the centers of reduction were required to make the temperatures of the vertical system and of the horizontal system interlock harmoniously and agree together on the sea-level plane. Furthermore, new and more accurate temperature gradients in latitude and longitude could now be obtained, and the work was therefore repeated

from the beginning to the end with the improved values. The adopted temperature system is the result of two or three such approximating computations, so that it has at last sufficient reliability to become the substantial basis for further reductions. The sea-level temperatures at the several stations can easily be scaled from these charts to the tenth of a degree, and such values are called t_s . The use of centers of reduction commends itself by the fact that the stations can be grouped in several ways, since the same station can be reduced to different centers, and the local inaccuracies will thus check themselves out; also by the fact that the entire amount of computation is much smaller and its accuracy can be controlled by the algebraic differences for uniform spaces. The most important result of this discussion is the development of well defined temperature inversions during the winter on the northern Rocky Mountain slope, and in the summer in the southern California districts. The former are due to the dynamic heating of the air blowing eastward over the Rocky Mountain divide, and the latter to the excessive surface heating of the arid region relatively to the temperature of the Pacific Ocean. The introduction of these inversion gradients relieves the congestion of the isothermal lines heretofore drawn in these districts.

THE FIRST PRESSURE REDUCTION TO SEA LEVEL.

Finally, the relative humidities were assumed to be the same for the surface and the sea-level plane throughout the plateau, and from the values of t_s just found the corresponding sea-level vapor tensions, e_s , were computed. We have thus obtained all the elements required for a reduction of the surface pressure, B , to the sea-level pressure, B_s , by taking as

a first approximation $\theta = \frac{t + t_s}{2}$ and the ratio $\frac{e_s}{B_s}$, where B_s is

nearly 30.00 inches for all monthly means. Using our new logarithmic tables, the monthly and annual pressure at each station in the United States and Canada was reduced to sea level, and the results were transferred to charts. Isobars were drawn through these sea-level pressures as accurately as the data permitted, though the values of B_s were quite discordant in many places, and the lines somewhat in doubt. Of course the plateau correction was included in the sea-level reduction, as stated above.

TO FIND $t - \theta$.

For practical working by the tables, it was first necessary to determine the relations of t and θ for the entire range of temperatures throughout the year, and this was a task of no little perplexity. It was, however, finally accomplished by two processes. It will be remembered that in the Abbe-Upton system of monthly constants and in the Hazen empirical tables this modification of the surface temperature argument was omitted; that Ferrel used a constant vertical gradient of 0.165° per 100 feet for the year to pass from t to θ , and that Morrill modified this gradient by taking per 100 feet, 0.150° in winter, 0.200° in spring and autumn, and 0.250° in summer. My vertical temperature gradient came out about 0.195° for each month in the year, as the average for the entire plateau, but it was distinctly shown that the several portions of the plateau have very different gradients in the same month, and that for the same locality they change greatly from month to month. Hence it was improper to attempt to deal with the plateau as a whole by using the same temperature gradient; so that, in fact, each station must be considered not only by itself, but also in its relations to the neighboring stations. Finally, special curves have been constructed for temperatures between -40° F. and $+100^\circ$ F., showing the variable difference between t , the surface temperature for twenty-four hours, and the corresponding θ , or the mean air temperature of an air column substituted for the plateau itself. The θ can not

be considered as the arithmetical mean temperature between the surface and the sea-level temperatures, because the connecting line is a curve and is not straight, so that it is essential to arrive at an integral mean temperature instead of an arithmetical mean. In a graphical construction the values of θ may be taken as the abscissæ and the differences, $t - \theta$, as the ordinates of a curve, which we seek to construct. The first approximation is evidently equal to $t - \theta = t - \frac{1}{2}(t + t_0) = \frac{1}{2}(t - t_0)$, but the true value may differ from this by several degrees at many of the high stations.

THE FIRST PROCESS.

We proceeded to discuss this point by two distinct methods, the first covering the low temperatures from -40° to $+30^\circ$, and the second covering the temperatures from about 10° to 90° , so that there shall occur a small overlapping of the two systems in the middle temperatures, and thus allow the two to be joined together. About fifty maps were selected for the winter season, when high pressures and low temperatures prevailed in the Rocky Mountain districts. The pressures for the plateau stations were next reduced to the 3,500-foot plane, because this requires the least average run for the corrections, and hence there is little error arising from selecting the wrong temperature arguments. This configuration of isobars was drawn in red lines; then the low stations near the Pacific Ocean and those in the Mississippi Valley were reduced to sea level; also some of the stations on the mountain slope at moderate elevations were reduced to the 3,500-foot plane as well as to sea level. A set of isobars was drawn on the sea level in blue lines. It was now assumed that the configuration on the 3,500-foot plane is substantially correct for that elevation, and is what the forecaster really wants at sea level for practical work. It was therefore joined with the sea-level system by simply making the red and blue lines flow together and uniting them smoothly; in other words, the upper configuration was depressed to sea level by simply renumbering the isobars in inches as determined by the true sea-level lines, so that a single system of well-balanced isobars covered the country. Next the question was, what is the value of θ that will be required to transform the observed station pressure into the sea-level pressure thus constructed? This was computed from the data in a reverse direction, and the differences, $t - \theta$, found; these were collected by groups for each station on the plateau above 1,000 feet in elevation; the means were taken and plotted as ordinates on the abscissa axis of θ . The result was very instructive, and it at once separated the plateau into groups corresponding to the geographical and climatic location, and showed that all the attempts to use one value of the vertical gradient for a given time are very erroneous. It should be remarked that the value of $t - \theta$ thus found was much too large, because it included within itself the real plateau effect, and this ought first to have been separated; but it gave true relative variations of $t - \theta$ with the range of temperature from -40° to $+30^\circ$, so that it was only necessary to discover the reduction factor to make the scale of values correct.

THE SECOND PROCESS.

For the warmer temperatures of the year, from $+30^\circ$ to $+90^\circ$, I took the mean monthly values of t and t_0 , surface and sea-level temperature, respectively, and found $t - \theta = \frac{1}{2}(t - t_0)$ and $\theta = \frac{1}{2}(t + t_0)$. These were plotted month by month in coordinate points through which it was easy to draw approximate mean curves. It is noted that during the winter months the ordinates average a little larger for the same values of θ than during the summer months, but as we are limited to constructing a set of tables representing mean conditions, this mean line is the best that can be taken. The variation on the mean line does not often exceed $\pm 1^\circ$, and this small change in the resulting argument has really but little influence upon the sea-level reductions which are required. Finally the slope of

the second system of curves at the temperatures from $+10$ to $+30^\circ$ indicated the slope that should be assigned to those found by the first method, that is, they gave us the scale factor for reducing the slope first obtained. The resulting curves are published in the full report, but they can hardly be described without diagrams. Generally speaking, on the north and east of the plateau the $(t - \theta)$ curves have a short ordinate from 10° to 40° , and a considerable increase toward either end; on the central portions of the plateau the curves are nearly flat, the length of the ordinates being about proportional to the altitude; on the western side of the plateau the curves have ordinates which are longest in the central parts and shortest at the ends, that is to say, they are about reversed in shape from those on the eastern plateau. These differing results are largely due to the climatic effects of prevailing winds from the Pacific Ocean, which blow upon the mountain ranges and precipitate their moisture on the western side; the clear skies and cold waves prevail on the eastern side; also there are seen to be certain dynamic heating effects. This subject is, however, too large to expand in this connection.

THE SECOND PRESSURE REDUCTION TO SEA LEVEL.

Equipped with these first approximate values of θ for each month as derived from the surface t , the reductions to sea level were made for the mean monthly normal station pressures, B_0 , as already mentioned, and the corresponding isobars were drawn. The sea-level pressures, as shown by the resulting map itself, apart from the reduced values, are really more nearly well balanced and correct than those derived from the individual reductions, because the isobars depend upon the mean result of many neighboring stations, whose mutual claims must be simultaneously satisfied in drawing the pressure lines. The pressures were, therefore, scaled from the maps, giving B_m , and the differences taken between them and the original values as reduced by the computation for $B_0 - B_m$. The outcome was exceedingly valuable and suggestive. For some stations the differences between the map and reduced values were such as to indicate only minor irregularities of a few thousandths of an inch, and these are to be referred to imperfections in the station normals; for others the difference was nearly constant, suggesting an error in the assumed elevation, especially for the old stations at military posts where the elevation had been derived from barometer readings; for others there was a very marked annual period in the differences, which could only be due to an error in the assigned value of the mean temperature, θ , since the differences disappeared at certain points, the signs being reversed between the low and the high temperatures. To be brief, all these sources of difference were removed, the entire work was recomputed a second time, a new system of isobars was drawn, and generally the entire subject was worked over in every available way. The practical effect was a readjustment of some elevations, and of the values of θ , so that the final differences between the map and the reduced sea-level pressures became small, usually less than one hundredth of an inch (0.010 inch), for the long record stations. In a few cases it was found that the constant error, called ΔA , was due to the fact that the initial temperature from which the plateau correction was computed, namely, $C \Delta \theta H$, was not accurately chosen. Usually this was taken as a mean annual temperature, but for some stations, especially on the southwestern edge of the plateau, Santa Fe, Flagstaff, Modena, Independence, etc., it should have been somewhat different. The variation can not be due to elevation, because this has been carefully determined by the surveys, but it must be caused by the local influence of the great desert in connection with the adjacent lofty mountain ranges. There are other stations of low elevation, lying in the eastern or in the Pacific coast districts, where no important error can arise from the reduction data, at which there is a small constant correction required to make the station harmonize with the others, as, for example, Lynchburg, Va., and

Portland, Me. These stations have been known, at the Central Office, to act out of perfect harmony with their surroundings, and it is still difficult to understand the causes of these discrepancies. It has been found, furthermore, that the low stations on the north Atlantic and south New England coast and also on the north Pacific coast, are not so perfectly in accord as might be expected, and this may be due to the effect of some land and sea action which is operating in these localities. On the whole the reductions as completed are very reliable when all corrections are applied, that is to about 0.010 inch, under all possible circumstances. We note further that the differences outstanding between the finally adjusted reductions to sea level from the station normal pressures and the map pressures derived from the balanced system of isobars, can be properly considered as corrections to the station normals which will reduce them to the homogeneous or balanced normals. This is distinctly true for stations of short record, e. g., two or three years, where the monthly variations are really considerable, so that by applying these residuals as corrections the station normals are brought to agree with the more correct system which would be derived from a long record of observations. In short, since the long record stations really control the map construction, the short records can be at once improved by applying these small final residuals. Such residual corrections have, therefore, been added to all station normals, and the entire system is thus reduced to a long range homogeneous system and it is called B_n , normal pressure at the station, and B_m , normal pressure at the sea level. These values become our standard normals for further developments and have been so used in the remainder of the work. It is also evident that whenever a new station is opened, we can easily compute a more correct station normal pressure, by starting with the values of B_m as interpolated from the map, than could be found by less than fifteen or twenty years of observations.

PRESSURES COMPUTED ON THE 3,500-FOOT AND THE 10,000-FOOT PLANES.

We have now obtained the following quantities: At the stations, B_n , t , e , R , H , normal pressure, temperature, vapor tension, and relative humidity; on the sea-level plane, B_m , t_o , e_o ;

also the ratio $\frac{e}{B}$ was computed for use in the reductions. It

is next proposed to compute B_1 , t_1 , e_1 , on the 3,500-foot plane, and B_2 , t_2 , e_2 , on the 10,000-foot plane. For this purpose the temperature gradients in the free air must first be determined. There are three sources of information available, namely, the European balloon ascensions, the American kite ascensions, and the Washington gradients derived from computation on the cloud formations observed with the theodolites in 1896-97. These were all thoroughly discussed and they agree together sufficiently well to permit the assignment of average gradients from the surface to the two upper planes in the free air. The temperatures were computed on these planes for enough stations to permit drawing systems of isotherms with accuracy. As regards the 3,500-foot plane, the temperatures were found from the free air gradients for stations outside the plateau and of lower elevation than 3,500 feet; for points within the plateau the temperatures on that plane were taken from the diagrams of vertical temperatures, previously constructed; these two systems agree well together, and the isotherms are continuous. The isotherms on the 10,000-foot plane are simple curves joining the Atlantic and Pacific districts and present no trouble in crossing the plateau. There is one result of interest, however, at the surface of the plateau, which I call "gradient refraction." Within the plateau the vertical temperature gradient averages about 0.195° per 100 feet, and in the free air for the eastern districts about 0.300° up to 10,000 feet. Now it is evident that this plane is high enough above the plateau to escape the influence of the surface con-

ditions, and that it is in the midst of the rapidly drifting current of air whose direction is eastward, so that quite uniform temperature must prevail along the same parallel of latitude. Hence, it follows that by using the smaller gradient 0.195° to the surface of the plateau, larger values than 0.300° must be employed from the surface to 10,000 feet, if the average gradient is to be about 0.300° , such as it would be if the plateau were removed. Therefore at the surface of the plateau there is something like an abrupt change in the gradients which is similar to refraction. Finally, by means of the temperatures thus found and the relative humidities, assumed to be the same as for the surface stations, the vapor tension on the 3,500-foot plane was computed. For the 10,000-foot plane it was assumed that the relative humidity is 50 per cent of the surface amount at all places; this may be subject to criticism, but it is near the truth and the effect on the vapor tension of even considerable changes in the relative humidity would be unimportant at the low temperatures prevailing at that altitude.

THE FIRST COMPUTATION OF B_1 B_2 .

Instead of computing the values of t_1 , e_1 , and t_2 , e_2 , for the several stations at the outset, the work was much shortened by interpolating the values of all this data on selected points of the charts, namely, centers of reduction; that is, where the meridians 5° apart, 125° , 120° , 65° , cross the parallels 5° apart, 55° , 50° , 30° . On these centers of reduction the sea level B_m , t_o , e_o were also drawn from the charts, so that the data is complete for reducing the sea-level pressures to the higher planes. There are two objects gained by this method of discussion; (1) the work of computation is shortened very much; and also (2) the result affords an admirable check on the entire system of reductions, as will be seen by what follows. The pressures B_1 and B_2 on the 3,500-foot plane and the 10,000-foot plane, respectively, were computed by the logarithmic tables from the data thus obtained on the centers of reduction, and the corresponding systems of isobars were drawn. There now exists the same general harmony in these isobars as on the sea-level plane, and no further corrections are required. It is to be especially noted that in the plateau region the reductions from sea level to the upper planes were made by the same principles as if it had been a free air column, so that all plateau questions are laid aside.

THE SECOND COMPUTATION OF B_1 B_2 .

From the B_1 and B_2 charts the pressures belonging to all the stations were interpolated, so that the values of B_1 , B_2 , to be derived by a direct computation from the station data could be compared as a check. Meanwhile the several station reduction tables to the three planes had been completed, and as a final check the three values, B_o , B_1 , B_2 , were computed and compared with the values derived from the charts, as explained in the first process. The differences between the two sets of values for B_o , B_1 , B_2 were about the same on the three planes; they average about 0.010 inch, the majority being 0.000 or 0.010 inch, a few 0.020 inch, with occasional larger variations due to errors of computation readily detected, or to a local peculiarity, involving a slight readjustment of the corrections in the station tables. These checks, therefore, involved the three distinct parts of the entire discussion, since the process has been arranged practically in a circuit so as to pass from the station B_o to B_1 and B_2 by two separate routes, as described. Hence, (1) the processes of eliminating the plateau effect, and of computing the temperature arguments t and θ were successful; (2) the logarithmic tables and the numerical station tables are in agreement; (3) the charts are accurately drawn, and represent the observations with precision.

As the result of this discussion we have prepared charts for the United States and Canada, giving the monthly and annual normals of pressure, temperature, and vapor tension on the sea-level plane, the 3,500-foot plane, and the 10,000-foot plane,

also the relative humidity on the sea-level plane, i. e., 130 charts for these data. There are also charts of gradients of temperature in latitude, in longitude, and in altitude; and charts of pressure variations for a few selected hours referred to the mean of 24 hourly observations. Furthermore, the corresponding numerical values are entered in a summary table for all stations on the sea-level plane, about 265 in number; also for all the stations which were in use by the Weather Bureau, either in the United States, Canada, and the West Indies, at the beginning of the year 1900, or which have been opened for service since that date, making about 175 on the upper planes.

It has not been found necessary to revise any of the reductions to sea level since the tables were put in operation on January 1, 1902, showing that they bear the test of practical work at the hands of many observers. The station tables for

the upper planes will soon be tried, and an estimate made as to their value in increasing the accuracy of the forecast system of the Weather Bureau.

We conclude with the remark that the pressure observations and computations of the United States have been at last placed upon a strictly scientific basis, and that all the corrections required by theory will be systematically applied in the future, and the entire series from 1873 onwards will be kept strictly homogeneous. We shall, therefore, for the first time be ready to take up the problems of seasonal variation of the weather, the changes of the climate and crop from year to year, and also the true cosmical problems involved in the radiation effects of the sun upon the earth's atmosphere. Even if we do not ourselves succeed in resolving these questions, we shall have left this portion of the data in form for others to make reliable discussions.

II.—METHOD OF OBSERVING AND DISCUSSING THE MOTIONS OF THE ATMOSPHERE.¹

INTRODUCTORY REMARKS.

It has been suggested to me that it would be advantageous to many who are interested in the progress of modern meteorology, if the results of the observations on clouds, which were made by the United States Weather Bureau during the years 1896-97, could be put in a more compact form than was adopted in the original report.²

I am the more inclined to present anew some of my results because of the extensive use that has been made of the American observations generally in Dr. J. Hann's *Lehrbuch der Meteorologie*, 1901. In this judicious and comprehensive summary of the state of meteorology at the end of the nineteenth century, Dr. Hann has given very generous recognition to the contributions of the United States, including the Weather Bureau and the Blue Hill Observatory, to the advancement of meteorology. But more important than this, the views therein adopted regarding the theories of the circulation of the atmosphere in the general and the local cyclones, are fully in accord with the ideas set forth in my report on the cloud observations of 1896-97. It is apparent that meteorology is at last securing a set of principles founded on observations, which will supersede much that has been heretofore taught in this connection. It is therefore important to explain the results of the Weather Bureau observations of 1896-97 as briefly and simply as possible.

Taking a very general view of the present state of meteorology, it may be proper to classify the conditions as follows: The statistical side of the subject is being rapidly worked up, so that our knowledge of facts is relatively quite complete in climatology, and in the diurnal and annual periods of the various atmospheric elements, namely, pressure, temperature, vapor tension, and wind direction, in different parts of the world, so far as they prevail in the strata near the ground. But in the upper strata our knowledge of these elements is still very limited, though it has been considerably extended during the past ten years, by the cloud observations, and the balloon and kite ascensions. On the theoretical side of static meteorology it may be said that meteorological analysis is well advanced, as far as concerns the barometric relations of pressure to the height, the temperature and vapor tension variations, and the adiabatic thermodynamics generally. The practical extension and application of these formulæ to the upper strata is making fair progress, and is likely to result in very definite knowledge of the true state of the atmosphere throughout its extent. In dynamic meteorology, however, that is in the hydrodynamics of the atmosphere, affairs are in an unsatisfactory condition, and they can be reclaimed only by pursuing a sound policy regarding them. Looking over the entire field, one is surprised to find that but little has been done in the preliminary and the most necessary stages of this work, in order to make the dynamics of the atmosphere a practical scientific problem. It is wasting time to speculate on the mathematical analysis of the motions of the atmosphere till we know what the motions are, simply as a case of kinematics. In other words, the paths of motion of the average air currents should be systematically worked up all over the world, as the indispensable preliminary to this study. Of course the obstacle in the way of doing this is the invisibility of the air itself, and the labor of making any observations on its direction and velocity of motion much

above the ground. It was for supplying just this need that the international cloud observations of 1896-97 were instituted, and to it they have contributed a valuable amount of data.

Furthermore, there are the great physical problems connected with the absorption of the sun's radiant energy in the atmosphere, its separation into several kinds of energy—electric and magnetic energy, heat energy of the visible and invisible spectrum, and so on. Also there is the question to be answered as to the amount of the solar output itself, the variations from its mean value, how much and what kinds of energy are absorbed in the upper strata, and what in the lower. The circulation of the atmosphere in its details really goes back to these questions about which we know only a very little. Hence, in a word, the deficiency of modern meteorology is in the dynamics of the upper and middle strata of the atmosphere.

NOTATION AND COORDINATES.

It is not an exceptional fact in the history of science that in the first stages of its development meteorology should have grown up in a rather haphazard fashion, especially as it was dealing with a subject of popular interest, wherein many observers were concerned in getting observations of one kind or another without much regard to their ultimate use in mathematical analysis. As the result of this lack of purpose the confusion became so great between the methods of observing and recording in different parts of the world that when comparative studies were begun the difficulties arising from the want of homogeneity were seriously felt. In order to remedy this state of confusion the International Meteorological Committee have been laboring for years to introduce uniformity into the methods adopted by meteorologists. Much has already been accomplished, and yet there are at least two very important steps that remain to be taken. The first is to use only one system of measures, as the metric in place of the metric and the English; and the second is to conduct and discuss the observations in such a way that in their published form they shall be in perfect order to meet the requirements of the fundamental mathematical equations, either static or dynamic, as they are needed. At present meteorological observations are about evenly divided between the English and the metric systems of measures, since the former is in use in Great Britain, Canada, United States, south Africa, Australia, and India; while the latter prevails in Europe, Asia generally, Japan, north Africa, and South America. Thus it is necessary to translate the figures from one system to the other in preparing the data for the world and for cosmical problems; also two sets of reduction tables are required for all the elements, and two sets of constants in all the formulæ. The more lamentable defect occurs from the fact that the observations are made without relation to their final use in mathematical discussions involving the motions of the atmosphere. Indeed almost nothing has been done to give us the true vector components of motion in the observations, so that they shall be in form for immediate introduction into the equations. The standard equations have been presented by different authors in many equivalent forms, and in consequence the subject has been made unnecessarily complex and difficult for students. The entire body of fundamental equations in meteorology is not very large, but the amount appears to be much greater than it really is by reason of the manifold notations and symbols which have been employed. No more valuable reform could be instituted than that of causing the same physical quantity

¹ Reprinted from the Monthly Weather Review for February, 1902.

² Report on the International Cloud Observations, May 1 1896, to July 1, 1897, by Prof. Frank H. Bigelow. Report of the Chief of the Weather Bureau, 1898-99, Vol. II.

to be always represented by the same symbol. Thus, for example, barometric pressure B , pressure in units of force P , pressure in units of weight p , would put us in harmony with the leading works in hydrodynamics and thermodynamics; then absolute temperature T , thermometric temperature t , mean temperature of the air column θ , vapor tension e , maximum vapor tension E , absolute weight of vapor μ , weight of the unit volume σ , specific weight ρ , and relative humidity $R.H.$ For rectangular coordinates, displacements (x, y, z) s , velocities (u, v, w) q , accelerations $(\dot{u}, \dot{v}, \dot{w})$ f , angular velocities $(\omega_1, \omega_2, \omega_3)$; for cylindrical coordinates, radius ϖ , angle about axis of rotation φ ; for polar coordinates, radius vector r , polar distance θ , angle about axis of rotation λ .

I have felt the weight of these considerations in my comparative studies so much that special pains have been taken in my report to exhibit all the fundamental equations in a standard system of notation, and also to reduce the analyses of several authors to the same standard system for the sake of ready intercomparison. Also, as it seems to be of the utmost importance that the observations taken to determine the motions of the atmosphere should be made in a form appropriate for use in the dynamic equations without further transformation of the data, particular care was taken to make the cloud observations conform to these requirements. It was not possible to bring about this harmony between observations and the analytical theory without introducing some radical changes in the methods heretofore followed by meteorologists, both in the conduct of the observations and in the analytic development of the equations. Accordingly, some account of these changes, as well as of the new results which were deduced from the cloud observations made by the United States Weather Bureau in 1896-97, will be given in the following pages.

THE AXES OF COORDINATES.

The first decision that must be made in establishing a fundamental system of notation has regard to the choice of the axes of coordinates which shall be placed at the base of the entire study, since all the algebraic signs of the quantities depending on the observations which are to be substituted in the equations, must be determined from the adopted positive direction of the axes. This choice depends practically upon two facts, (1) that the radius of the earth is drawn positive outwards, since r increases from the center, and (2) that the right-hand rotation is adopted generally in modern scientific researches. It is true that some of the German mathematicians use the left-hand rotation, but the trend is toward a universal adoption of the right-hand system. If we take as the primary radius that of the earth's axis of rotation in the Northern Hemisphere, as is most appropriate for all scientific problems except in terrestrial magnetism, then in polar coordinates the positive angular development in polar distance, θ , is southward; next, with the right-hand rotation about this axis extended upward, the positive development of λ , the angle in longitude, is eastward. Hence, for all systems of coordinates, polar, cylindrical, and rectangular, the azimuth rotation is *from the south through the east, north, and west*. Unfortunately this is in the opposite direction to the azimuth rotation adopted in astronomy, in navigation, and in popular meteorology, because in these branches of science the simple practical consideration has been to follow the sun in its diurnal course, so that azimuth circles and compass cards are numbered around in the clockwise or left-hand rotation. For many statistical purposes, as where average wind directions are to be computed, it makes little difference what system of notation is used, because these data do not look beyond their own immediate purposes. But where we have to deal with a system of equations it is not so. If we take the first set of equations, International Cloud Report (154), for linear velocities due to rotation,

$$\begin{aligned} u_1 &= u - y \omega_3 + z \omega_2 \\ v_1 &= v - z \omega_1 + x \omega_3 \\ w_1 &= w - x \omega_2 + y \omega_1 \end{aligned}$$

where x, y, z are linear distances, u, v, w are linear velocities, $\omega_1, \omega_2, \omega_3$ are angular velocities, it is evidently necessary that all these should be defined most carefully. According to the statement given above,

- + (x, u) are referred to the southward axis,
- + (y, v) are referred to the eastward axis,
- + (z, w) are referred to the zenithward axis,

but everything will go wrong if ω_3 is not taken to rotate positively about the axis z , from the south through the east, instead of through the west. Hence, ω_1 turns the axis of y to z , ω_2 the axis of z to x , ω_3 the axis x to y , in cyclical order. Thus it happens that, having adopted this system of rotation, it was found necessary to transform the equations of some well-known mathematical papers in dynamic meteorology to agree with it.

THE AZIMUTH ROTATION.

There is one further difficulty to overcome in regard to the popular meteorological system. An observer determines the direction of the wind by looking toward it and feeling the force of it on his face, or he sets up a wind vane with an arrow pointing in the direction *from which* it blows. This is, however, exactly contrary to the method that mathematical physicists employ, for they describe a stream line by the direction *toward which* it flows. An arrow is drawn on a map "down stream" to show how the current flows, or on the weather map an arrow is said to "fly with the wind." In the latter case meteorologists are inconsistent with themselves, but they adopt the correct principle in their precept on the map. If we describe a wind as having a velocity of so many miles per hour from a given direction, say the north, this must be changed in azimuth through 180° to the south in order to be of use in analytic work. We have thus to make *two reversals* in the common meteorological system: (1) the wind vector must be turned through 180° , and (2) the azimuth must be numbered from the south = 0° toward the east = 90° , north = 180° , and west = 270° . These two changes render it impossible to use the ordinary wind records which are found in meteorological reports without making this transformation. While it is not a very important matter for a few individual cases, it becomes a serious task to do the work of remodeling the figures for a large amount of data. It was for the purpose of saving this labor that the observations of the Weather Bureau were executed in 1896-97 on the correct system, so that all the figures appearing in the final report should be at once ready for use in the equations of hydrodynamics.

In order to facilitate the understanding of the discussion that follows, a chart is introduced to show the scheme of the operations. Fig. 1 gives a comparison of the azimuth system commonly employed by meteorologists with the one adopted in this report. The former is the left-hand rotation, such as astronomers use, and is counted from the south through the west point of the horizon. In this system, also, the observer faces the wind and gives the azimuth of the direction *from which* it blows. This has been abandoned for the reasons already mentioned, and in place of it is substituted the right-hand rotation, wherein the azimuth is counted from the south through the east point. Here the observer receives the wind on his back and looks toward the direction in which the arrow flies and *toward which* the air is moving. Thus a wind *from the NW* by the former system, with azimuth angle 135° , becomes a wind *toward the SE*, with the azimuth angle 45° , by the latter system. The working out of the results by this system, ready for analytic discussion, as will be seen, involves a minimum of computation, and besides this it reduces all the velocity components to the fundamental rectangular system of

coordinates adopted by Ferrel in his treatise, and continued in my "Standard System" which forms a part of the Report.

FIG. 1.—Comparison of the two azimuth systems.

Left-hand rotation gives azimuth of "motion from."
Right-hand rotation gives azimuth of "motion toward."
Positive translation is vertically upward.

The Marvin nephoscopes, with which the observations were made, were all graduated to read in a right-hand (anticlockwise) azimuth; the theodolites were also read in the same direction, and the azimuths of Tables 6 and 9 of the International Cloud Report are, therefore, in accord with the coordinate directions of the formulæ which are developed in the following portions of the same Report.

THE COMPOSITION AND RESOLUTION OF THE VECTORS OF MOTION.

It is sometimes necessary to construct the resultant velocity and direction of the motion of the air at a given place out of a large number of individual observations, as in forming charts 20-35, International Cloud Report, for example, and the following practical devices were found convenient. Suppose it is desired to determine the velocity and direction of motion in the cumulus cloud level on all sides of a low area, as in the Mississippi Valley. We can best proceed as follows: Take a piece of tracing paper, and select a large number of cloud maps showing about the same configuration of the isobars, so that the centers of the cyclones are located in a given district. Then lay the paper on the cloud maps in succession, and trace the arrows showing the cloud motion wherever an observation is found on the map. Mark the center on the first map and preserve it so as to place it in coincidence with the other cyclonic centers. Continue to fill the paper till some such composite of arrows is obtained as is shown within the square of fig. 2. A scale map of squares, or any other adopted division of areas, is to be prepared as large as the tracing paper, and the two are placed together so that the scale diagram marks off the arrows of the composite map into groups, within each of which it is proposed to find the resultant. Then count out the number of arrows pointing N, NE, E, etc., in succession for eight directions, giving in our example, N=4, NE=4, E=10, etc.; take the excess in four directions, as S=7, E=6, SE=7, SW=10; plot these results on a diagram and resolve SE=7 into E=5 and S=5; also SW=10 into W=7 and S=7; make the sums E=+4 and S=+19; plot these compo-

nents and obtain the resultant $V=20$; the angle $\varphi=9^\circ$ can be found by the use of a circular protractor, or it can be computed by the formula $\tan \varphi = \frac{E}{S}$, having regard to a change of algebraic signs for W=-E and N=-S.

Wind vector.	Component.	
	E	S
S=7		+7
E=6	+6	
SE=7	+5	+5
SW=10	-7	+7
	+4	+19

$V=20$; $\varphi=9^\circ$, that is, = S 9° E.

$$\tan \varphi = \frac{E}{S} = \frac{4}{19} = 0.21.$$

Fig. 2.—Example of the graphic composition of wind vectors.

It is easy to perform a large amount of graphic composition in a short time after a little practise, by arranging this work systematically. In the collection of vectors from the maps the total number will differ from square to square, and it is necessary to reduce the resultant to a common standard number. Suppose we adopt 40 arrows as the standard, then the completed resultant velocity must be reduced in that proportion.

Our example contained 64 arrows, hence $20 \times \frac{40}{64} = 12$, and 12 is to be adopted as the average velocity of the motion in the azimuth 9° , that is, S 9° E. These resultants assume that the average of a number of observed directions gives a relative velocity of motion, which can be reduced to an absolute velocity as soon as the true mean motion is determined from some other

source, as by theodolite observations. Charts constructed in this way are quite correct as to the direction of the motion of the atmosphere, and they give the relative velocities in different parts of a cyclone or anticyclone with sufficient precision to permit further important studies.

By the nephoscope observations the actual velocities in different portions of the area surrounding the center of motion were computed and collected for the several subareas about the highs and lows, as explained in Chapter 7 of the International Cloud Report. The relative velocities there recorded can be turned into actual velocities by utilizing the corresponding theodolite observations. The nephoscope refers all the observed motions to the 1000-meter plane, and the theodolite to the actual plane of motion at the height given by the angular measurements. I note that Dr. J. Hann, in his *Lehrbuch der Meteorologie*, pages 272, 273, and 275, attributes certain cloud heights of the Weather Bureau observations to the nephoscopes, but this is a mistake, because all our heights were determined by the theodolites. The heights associated with the nephoscope observations were adopted for translating the relative velocities of the nephoscopes into actual velocities, and his impression doubtless arose from printing such adopted heights in conjunction with the other data which were derived from nephoscopes.

THE RESOLUTION OF FORCES.

In the study of the motions of the atmosphere at all levels there are two types of resolution of vectors to be provided for in the discussion, the first in rectangular coordinates,

$$\begin{aligned} +x &= \text{South, } +y = \text{East, } +z = \text{zenithward,} \\ -x &= \text{North, } -y = \text{West, } -z = \text{nadirward,} \end{aligned}$$

in order to apply them to the motions of the general circulation over the entire hemisphere; and the second in cylindrical coordinates,

$+x$ = radial outward,
 $-x$ = radial inward,
 $+y$ = tangential counter-clockwise,
 $-y$ = tangential clockwise,
 $+z$ = vertical upwards on the axis,
 $-z$ = vertical downwards on the axis, the results being used in the analysis of cyclones and anticyclones, that is in the local circulations. It is evident that the vectors provided by the theodolite and nephoscope observations, in the form V = velocity and φ = azimuth counted from the south through the east, are ready for simple trigonometric resolution into the velocity coordinates (u, v) in the four quadrants by using the proper signs. When all the vectors (V, φ) are resolved in the north-south and west-east directions, we can take the mean values by summation and then compute the average motion of the entire mass of air circulating at a given altitude over any locality. It is necessary to obtain these mean directions of motion for the entire circulation, in order to be able to resolve out the special components of local circulation belonging to the cyclones and anticyclones. In the report on the International Cloud Observations the results of this rectangular resolution of the observed mean vectors are set forth in Table 33 as a summary; in Tables 34, 35, and 36 for high areas in the northern and southern portions of the United States; in Tables 38, 39, and 40 for the low areas in the northern and the southern portions of the United States; in Tables 42 and 43 the component southward and eastward velocities in highs and lows, also in selected areas; and in Tables 48, 49, 50, and 51 the seasonal velocities in the upper and lower cloud levels. This data bears directly upon the problem of the upper air currents in the general circulation, and similar data ought to be obtained in all portions of the world.

These rectangular, meridional, and longitudinal velocity components are marked u, v , in order to distinguish them from

the cylindrical components which are designated u, v . Having constructed the individual components of general motion u, v in all the subareas together with the corresponding normal velocities, it is evident that the algebraic differences between them gives the true cyclonic and anticyclonic components, still in rectangular coordinates, as in Tables 44 and 45. The next step is to transform them into cylindrical coordinates u, v with the least possible labor. For this purpose it is important to select the subareas surrounding the center of a local circulation in such a manner as will contribute to that purpose. If the areas marked out by the 5° meridians and parallels of latitude are taken, it is impossible to transform the component velocities without a most tedious computation. Such areas are suitable for a simple display of the stream lines, but they do not readily lend themselves to the composition and resolution of vectors of motion.

I have, therefore, adopted the following plan for the subareas surrounding a center of motion. They each have about the same relative area, and they are distributed as far as possible on the cardinal lines of the compass direction. They are numbered with the right-hand rotation, and they are central each on some cardinal point of the compass. Thus on the north-south line 17, 9, 3, 1, 5, 13 are located; on the west-east line 19, 11, 4, 2, 7, 15 are found; while 18, 10, 6, 14 run from NW to SE, and 20, 12, 8, 16 from SW to NE. Hence it is seen that all the N-S and W-E areas are immediately available for composition in rectangular and in cylindrical coordinates, while the rectangular coordinates of the NW-SE and SW-NE areas which come from the first collection need only a simple transformation to become the radial and tangential components of cyclonic circulation. These latter are to be ultimately worked out, and we shall then have three sets of coordinates arranged symmetrically about the center

FIG. 3.—Plan of the subareas, azimuths, and compass points, adopted in high and low areas, for the discussion of cloud observations.

The mean of areas 1 to 4 = I, at the average distance 250 km.
 The mean of areas 5 to 12 = II, at the average distance 750 km.
 The mean of areas 13 to 20 = III, at the average distance 1,250 km.

of the three circles I, II, III at certain evenly distributed distances. The scale of the original diagram, Chart 9, International Cloud Report, is on a radius of 3 centimeters; on the weather maps this is equivalent to 15 centimeters, where 1 centimeter is equal to 100 kilometers. The adopted scale is therefore *one-fifth* the scale of the daily weather map, and on it 1 centimeter represents 500 kilometers or 310.7 miles, and one millimeter is equivalent to 50 kilometers, or 31.1 miles. All the diagrams of the Report, as far as possible, are reproduced on this scale, but they are readily interpreted on the weather map, so far as linear dimensions are concerned.

VECTORS OF MOTION IN HIGH AND LOW AREAS—RECTANGULAR COORDINATES.

In order to prepare the observations for discussion all those

which were made in the same subarea of a cyclone or an anticyclone were collected together in each cloud stratum, and the resultant of all these individual vectors was computed in accordance with the method above described. The individual observations occur in Table 9, and the mode of collecting them is illustrated in Table 29, page 363 of the Report. For convenience, the United States was divided into six districts: 1, Alberta; 2, Lakes; 3, New England; 4, Colorado; 5, West Gulf; 6, South Atlantic; so as to arrive at a conception of the prevailing local characteristics. Hence the heading of the form H-2-15, occurring in several tables, means that in subarea 15, of a high area or anticyclone whose center is in the Lake district, the accompanying observations were made in the several cloud strata, and also at the surface where the instrumental meteorological data are given at the three daily observations. Table 32 contains the resulting vectors V, φ for the northern and southern groups by districts, also for the four seasonal quarters of the year, together with the several mean values, all this extending to the eight cloud strata. In this table the relative velocities are given as derived from the nephoscope, that is on the 1,000-meter plane.

TABLE 1.—Direction and velocity of motion in high and low areas—rectangular coordinates.*

Compass Point.	Area number.	Cirrus; average height 9.8 kilometers.					
		High.			Low.		
		No.	φ	V	No.	φ	V
S	1	25	86	35.3	7	111	51.0
E	2	16	57	43.1	4	96	40.2
N	3	49	76	31.4	...	...	...
W	4	30	78	36.3	6	104	58.8
S	5	37	66	37.2	50	97	31.4
SE	6	20	67	32.3	33	101	44.1
E	7	23	90	58.8	10	109	36.3
NE	8	43	89	29.4	...	...	...
N	9	51	81	34.3	7	90	49.0
NW	10	34	117	38.2	3	92	27.4
W	11	42	80	37.2	23	77	45.1
SW	12	12	107	14.7	6	105	56.8
S	13	38	95	31.4	27	123	33.3
SE	14	64	82	35.3	70	93	32.3
E	15	28	100	46.1	49	85	34.3
NE	16	58	85	28.4	24	90	30.4
N	17	27	88	36.3	...	...	...
NW	18	25	96	33.3	2	122	39.2
W	19	51	98	29.4	36	71	44.1
SW	20	63	109	30.4	94	90	41.2
No. of obs....		736	...	...	451	...	...
Means		...	...	34.9	...	...	40.8

* Extracts from Tables 34 and 38.

The vectors of Table 32 are plotted on Chart 13 of the Report, the northern in red and the southern in blue, first for the high areas and then for the low areas; also in the seasonal groups so that the comparative motions can be studied. It is evident that several years work are needed to produce smooth and evenly balanced mean vectors, which shall truly represent the average circulation. Especially it will be necessary for the Canadian stations to cooperate and supply the vectors wanting in the northern subareas of our three northern districts, as this part of the circulation usually extends into Canada. Furthermore, the vectors of Table 32 are collected together numerically in Tables 34 to 40, with a single change, namely, that the velocities observed on the 1,000-meter plane have been multiplied

by the adopted mean height of the given cloud stratum. For example, the mean height of the cirrus is taken as 9.8 kilometers, and hence the mean annual velocity $V = 3.6$ of the cirrus in high area No. 1, page 368, is multiplied by 9.8, and it is entered at the beginning of Table 34 as $V = 35.3$. I have taken the cirrus in each subarea of the high and low areas to show as an example in Table 1 and fig. 4.

FIG. 4. (From Chart 15.)

These vectors are plotted on Chart 15 (see fig. 4), which shows the annual vectors on the several cloud levels in high and low areas. From Chart 15 and the Tables 34-40 are obtained the data for discussing the mean general circulation over the United States. The mean total velocities in high and low areas, without regard to direction, are found by taking the mean of the velocities in the areas of Tables (34-40) and they are given in Table 33, section 1, as in the following example, Table 2:

TABLE 2.—Total velocities in highs and lows without regard to directions.*

Clouds.	High areas.				Low areas.		
	Height, kilom.	All groups.	North-ern.	South-ern.	All groups.	North-ern.	South-ern.
Ci.	9.8	34.9	38.3	30.4	40.8	44.6	28.3
Ci. S.	9.8	39.1	42.6	34.8	39.8	42.5	36.3
Ci. Cu.	8.1	33.5	33.9	30.5	39.3	43.8	34.8
A. S.	5.9	30.2	31.1	24.1	36.0	39.4	30.5
A. Cu.	4.5	23.5	26.6	19.7	29.2	32.6	24.4
S. Cu.	2.5	23.3	22.7	18.5	28.6	32.9	21.1
Cu.	1.5	11.2	10.9	10.4	14.6	17.4	11.8
S.	0.9	11.4	12.2	9.5	11.1	13.2	8.6
Wind.	0	4.8	4.9	4.8	5.4	5.3	5.9
Range, per ct.				19	15		28
From Table.		34	35	36	38	39	40

* Extract from Table 33, Section I.

Table 2 shows that the velocities are greater in the northern circuit than in the southern, and greater in the low areas than in the high areas. These values must be studied in connection with the barometric gradients to form a theory of the dynamic action in the atmospheric circulation.

The vectors in the form velocity and azimuth, V, φ , are next resolved into rectangular components in the north-south, west-east direction by the trigonometric rules, and the results are given in Tables 42, 43, from which the example in Table 3 is taken.

The means of these components are taken out in two ways: (1) the algebraic mean which gives the rectangular coordinates of motion in the general circulation for the highs and the lows, respectively. The mean of these last forms the normals from which the true cyclonic components are computed, and they are printed in heavy faced type. These results are collected together in Table 33, Sections II and III, from which the extract in Table 4 is made.

TABLE 3.—General rectangular components of motion in high and low areas.*

Compass point.	Area number.	Cirrus, average height 9.8 kilometers.			
		High.		Low.	
		S +	E +	S +	E +
S	1	+ 2.5	+35.2	-18.3	+47.7
E	2	+23.5	+36.2	- 4.2	+40.0
N	3	+ 7.6	+30.5		
W	4	+ 8.6	+35.5	-14.2	+57.0
S	5	+15.1	+34.0	- 3.8	+31.2
SE	6	+12.6	+29.7	- 8.4	+43.3
E	7	0.0	+58.8	-11.8	+34.3
NE	8	+ 0.5	+29.4		
N	9	+ 5.4	+33.9	0.0	+49.0
NW	10	-17.3	+34.0	- 1.0	+27.4
W	11	+ 6.4	+36.6	+10.1	+43.9
SW	12	- 4.3	+14.1	-14.7	+54.9
S	13	- 2.7	+31.3	-18.1	+27.9
SE	14	+ 4.9	+34.9	- 1.6	+32.3
E	15	- 8.0	+45.4	+ 3.0	+34.2
NE	16	+ 2.5	+28.3	0.0	+30.4
N	17	+ 1.3	+36.3		
NW	18	- 5.2	+33.1	-20.8	+33.2
W	19	- 4.1	+29.1	+14.4	+41.7
SW	20	- 9.9	+28.8	0.0	+41.2
Means		+1.97	+33.7	-5.26	+39.4
Normals		- 1.6	+36.6	- 1.6	+36.6
Means		+0.66	+40.1	-3.75	+32.7

* Extract from Tables 42 and 43.

TABLE 4.—Southward and eastward components of velocities in highs and lows.*

Clouds.	High areas.		Low areas.		Means.	Means.
	+S -N	+E -W	+S -N	+E -W	North.	East.
Ci	+ 1.97	+33.7	- 5.26	+39.4	- 1.6	+36.6
Ci. S	+ 1.65	+32.0	- 9.24	+35.9	- 3.8	+34.0
Ci. Cu	- 0.60	+32.6	- 3.00	+37.2	- 1.8	+34.9
A. S	- 0.37	+27.2	- 4.60	+31.3	- 2.5	+29.2
A. Cu	- 0.07	+22.1	- 2.38	+24.3	- 1.2	+23.2
S. Cu	- 0.32	+16.0	- 4.00	+24.3	- 2.2	+20.2
Cu	- 0.13	+ 5.1	- 0.11	+11.4	- 0.1	+ 8.3
S	- 1.22	+ 5.8	- 1.32	+ 7.8	- 1.3	+ 6.8
Wind	- 0.69	+ 1.1	- 0.40	+ 1.5	- 0.5	+ 1.3
From Table	42	42	43	43		

* Extract from Table 33, Sections II and III.

The most important remark to be made regarding these extracts is that the observations show an average northern component in the United States in all levels, provided it is a fact that as much air streams through the low areas as through the high areas on the average. (2) The subareas were collected into two groups, those having a southward and those having a northward component. Thus we have a southward component in high areas in 16, 8, 2, 7, 15, 6, 14, and in low areas in 18, 10, 4, 11, 19, 12, 20; but a northward component in high areas in 18, 10, 4, 11, 19, 12, 20, and in low areas in 16, 8, 2, 7, 15, 6, 14. The means from these groups give the mean

TABLE 5.—Component velocities in selected areas between high and low centers.*

Clouds.	Selected areas.			
	Southward.		Northward.	
	II. 16, 8, 2, 7, 15, 6, 14 L. 18, 10, 4, 11, 19, 12, 20		II. 16, 8, 2, 7, 15, 6, 14 L. 18, 10, 4, 11, 19, 12, 20	
Ci	+ 0.66	+40.1	- 3.75	+32.7
Ci. S	- 2.11	+36.9	- 3.89	+38.9
Ci. Cu	+ 4.95	+38.7	- 7.34	+32.1
A. S	+ 2.79	+26.5	- 7.47	+31.0
A. Cu	+ 6.24	+23.7	- 7.78	+21.9
S. Cu	+10.22	+22.1	-11.13	+17.1
Cu	+ 6.52	+ 9.6	- 8.13	+ 6.5
S	+ 5.25	+ 7.5	- 7.97	+ 5.1
Wind	+ 2.23	+ 3.2	- 3.25	+ 0.2
Range				
From Table	42	42	43	43

* Extract from Table 33, Section IV.

components of the distinctly southward and northward currents in the different strata. They are collected in Table 33, Section IV, and are reproduced in Table 5.

It is important to note that the most rapid currents, both northward and southward in the atmosphere, are in the strato-cumulus level, 2.5 kilometers or 1.6 miles above the ground, and that these currents decrease in velocity above and below that level. The eastward velocity averages about the same in the highs and lows. Hence we infer that the strato-cumulus level is the stratum where the interchanging motion is most rapid between the Tropics and the poles.

VECTORS OF MOTION IN HIGH AND LOW AREAS—CYLINDRICAL COORDINATES.

We can now compute the true cyclonic and anticyclonic rectangular components by simply subtracting the normal values (heavy type, Table 3) from the individual subarea values in each cloud stratum. In this way the components u_1 , v_1 of Tables 44, 45 are found, and an example is given above, in Table 6, in the cirrus level for the high and low areas. Against subareas 6, 8, 10, 12, 14, 16, 18, 20 there are placed the corresponding vectors (σ , β), velocity and azimuth, because these are needed in resolving the rectangular into cylindrical components. In the other subareas the components are already in the N-S and W-E directions and they can be transformed by mere inspection into the corresponding cylindrical coordinates, which are radial and tangential to a circle about the central axis, and they give u_2 , v_2 as in Table 6, and fig. 5 following, in which $+u_1$ = southward, $+v_1$ = eastward, $+u_2$ = radial outward, $+v_2$ = tangential anticlockwise.

FIG. 5. (From Chart 16.)

TABLE 6.—Anticyclonic and cyclonic components, cirrus 9.8 kilom.*
HIGH AREAS.

Area No.	Rectangular components.				Cylindrical components.	
	u_1	v_1	σ	β	u_2	v_2
1	+ 4.1	— 1.4			+ 4.1	— 1.4
2	+25.1	— 0.4			— 0.4	—25.1
3	+ 9.2	— 6.1			— 9.2	+ 6.1
4	+10.2	— 1.1			+ 1.1	+10.2
5	+16.7	— 2.6			+16.7	— 2.6
6	+14.2	— 6.9	15.7	333	+ 4.9	—14.9
7	+ 1.6	+22.2			+22.2	— 1.6
8	+ 2.1	— 7.2	7.5	286	— 6.6	+ 3.6
9	+ 7.0	— 2.7			— 7.0	+ 2.7
10	—15.7	— 2.6	16.0	190	+13.2	— 9.2
11	+ 8.0	0.0			0.0	+ 8.0
12	— 2.7	—22.5	22.8	263	+14.0	—18.0
13	— 1.1	— 5.3			— 1.1	— 5.3
14	+ 6.5	— 1.7	6.8	345	+ 3.4	— 5.9
15	— 7.4	+ 8.8			+ 8.8	+ 7.4
16	+ 4.1	— 8.3	9.2	296	— 8.7	+ 3.0
17	+ 2.9	— 0.3			— 2.9	+ 0.3
18	— 3.6	— 3.5	5.0	225	+ 5.0	0.0
19	— 2.5	— 7.5			+ 7.5	— 2.5
20	— 8.3	— 7.8	11.4	223	0.0	—11.4

LOW AREAS.

Area No.	Rectangular components.				Cylindrical components.	
	u_1	v_1	σ	β	u_2	v_2
1	—16.7	+11.1			—16.7	+11.1
2	— 2.6	+ 3.4			+ 3.4	+ 2.6
3						
4	—12.6	+20.4			—20.4	—12.6
5	— 2.6	— 5.4			— 2.2	— 5.4
6	— 6.8	+ 6.7	9.6	136	— 0.1	+ 9.6
7	—10.2	— 2.3			— 2.3	+10.2
8						
9	+ 1.6	+12.4			— 1.6	—12.4
10	+ 0.6	— 9.2	9.3	274	+ 6.1	+ 7.0
11	+11.7	+ 7.3			— 7.3	+11.7
12	—13.1	+18.3	22.6	127	—22.5	+ 3.2
13	—16.5	— 8.7			—16.5	— 8.7
14	0.0	— 4.3	4.3	270	— 3.0	— 3.0
15	+ 4.6	— 2.4			— 2.4	— 4.6
16	+ 1.6	— 6.2	6.4	285	— 5.5	+ 3.2
17						
18	—19.2	— 3.4	19.6	190	+16.1	—11.3
19	+16.0	+ 5.1			— 5.1	+16.0
20	+ 1.6	+ 4.6	5.0	70	— 2.2	+ 4.6

*Extracts from Tables 44, 46 and 45, 47.

Those which do not lie on the north-south west-east lines are transformed as follows: The coordinates u_1 , v_1 are compounded into the vector (σ, β) , σ being the linear distance from the center, and β the angle from the south. Thus in cirrus 6, high, Table 44, $u_1=14.2$, $v_1=-6.9$, and we find $\sigma=15.7$, $\beta=333^\circ$, which can be verified by reference to Chart 16 a. In the same way all the vectors under subareas 6, 8, 10, 42, 14, 16, 13, 20 which lie on the SE-NW and NE-SW diagonals have been reduced to vectors (σ, β) .

TABLE 7.—Mean components grouped in three levels.*
MEAN ANTICYCLONIC COMPONENTS.

		I.	II.	III.
Upper level. Ci., Ci. S., Ci. Cu.	u_2	— 3.3	+ 3.9	+ 2.2
	v_2	— 4.5	— 5.2	— 4.8
	σ	5.6	6.5	5.3
	β	234	307	294
Middle level. A. S., A. Cu., S. Cu.	u_2	0.0	+ 4.2	— 0.8
	v_2	— 7.1	— 6.6	— 9.3
	σ	7.1	7.9	9.3
	β	270	303	265
Lower level. Cu., S., Wind.	u_2	+ 3.3	+ 3.0	+ 1.8
	v_2	— 4.1	— 7.0	— 6.2
	σ	5.3	7.6	6.4
	β	308	294	287

MEAN CYCLONIC COMPONENTS.

		I.	II.	III.
Upper level. Ci., Ci. S., Ci. Cu.	u_2	— 1.2	— 6.8	— 1.8
	v_2	+10.2	+12.3	+ 0.7
	σ	10.3	14.0	2.0
	β	96	119	161
Middle level. A. S., A. Cu., S. Cu.	u_2	— 7.3	+ 0.3	+ 1.6
	v_2	+18.6	+14.4	+ 5.2
	σ	20.0	14.4	5.5
	β	111	89	73
Lower level. Cu., S., Wind.	u_2	+ 0.3	— 2.4	— 1.5
	v_2	+ 7.9	+ 6.3	+ 3.8
	σ	8.0	6.7	4.2
	β	88	111	112

* Copy of Table 52.

To reduce these to the cylindrical coordinates, we subtract 45° from β in 6 and 14, 135° from β in 8 and 16, 225° from β in 10 and 18, 315° from β in 12 and 20. Then the vector $[\sigma(\beta-\alpha)]$ is resolved at once into u_2 , v_2 . Thus 15.7, 333° of Ci. 6, high, Table 44, becomes 15.7, 288° ; thence $u_2=4.9$, $v_2=-14.9$, as in cirrus 6, high, Table 46, and as can also be verified on the Chart 16 a. In this way the coordinates of the anticyclonic and cyclonic components, Tables 46, 47, have been found. We have thus the data in such form that one more concentration can be made. If we assume that a symmetrical gyratory circulation is represented by the coordinates of Chart 16, it is now necessary simply to take the mean values of the cylindrical coordinates lying on each circle; that is to say, the mean of the areas 1-4, 5-12, 13-20, respectively. This has been done, and they are entered as I, II, III, in the next section of the same Tables 46, 47. The means of I, II, III, themselves, are also entered on the next line, as average values for the entire circulation in each cloud level. These can be most conveniently studied by reference to Charts 17, 18, where specimen vectors are plotted for the several levels.

The rectangular components were transferred to Chart 16 and drawn so that the vectors shall be central on the circles I, II, III, which run through the middle of the respective

adopted subareas. In order to get some idea of the average cyclonic and anticyclonic vectors in the different levels, the mean values of the vectors found on the circles I, II, III, respectively, were taken, and these give the relations between the inner and the outer portions of the masses of air in motion in cyclones and anticyclones. They are shown in Charts 17 and 18. To secure one more concentration of the data, and to further eliminate the local defects, the nine levels were reduced to three by taking the means of the three upper, the three middle, and the three lower strata together, respectively, and these are shown on Chart 19. The accompanying small Table 7 gives the corresponding numerical results; it is Table 52 of the cloud report.

It is evident that it would be of great advantage to meteorology to have similar observations continued systematically in

the United States, so as eventually to obtain perfectly reliable vectors of motion throughout the atmosphere, and they should be extended to all parts of the world as rapidly as practicable. It is not very safe to draw conclusions extending to the entire atmosphere from the observations made at a few selected localities, such as those in the United States or Europe, but it seems to be necessary for us to do so in the present incomplete state of meteorology. Moreover, we must use the material we now have in discussing what are the fundamental principles of dynamics that can be admitted into the theory, and accordingly I shall proceed to take up the observed general circulation and the local circulations, and compare them with the existing theories in order to arrive at such views as will probably determine the theoretics of the dynamic meteorology of the future.

III.—THE OBSERVED CIRCULATION OF THE ATMOSPHERE IN THE HIGH AND LOW AREAS.¹

GENERAL DESCRIPTION OF THE VECTORS OBTAINED BY OBSERVATION.

In my original report on the cloud observations of 1896-97, it was necessary to present the data in such a form that other students could have the facts at first hand. As then pointed out there are several subareas in which only a few observations were located, and they are quite unevenly distributed about the central axis, so that the final vectors as computed do not have the well-balanced smoothness which it is desirable to obtain. The data was given in the form of tabulations and also of diagrams, since it is easier to secure from the latter a clear mental picture of the average configuration of the vectors of motion in all parts of the cyclones and anticyclones. Having done this at the outset I now proceed to draw up an average system of vectors by the process of graphic adjustment. There will still remain some uncertainty as to the finer details in certain areas where the motion is more complicated, but I am quite sure that the results presented in this paper give a very correct idea of the mean motions of the atmosphere over the United States and Canada. It would require a good deal more labor in observation and computation than was involved in a single year's campaign to bring the work to that degree of perfection which is desired by meteorologists; this work must undoubtedly be expended in the interest of science some time in the future. Especially for the higher strata of the high and low areas do we need more observations, because the powerful eastward drift quickly obscures the comparatively small gyratory components that penetrate up to the high levels. It should be remembered that the vectors in hand were procured by observing the motions of the air almost daily throughout the year, and consequently that all kinds of weather have entered our final results. If we want the characteristic circulation pertaining to well developed cyclonic and anticyclonic configurations, it can be found only by selecting the vectors on certain days when these types are strongly organized, and discussing them by themselves. Under the circumstances that pertained to the cloud year we were obliged to put every kind of observation together, without selection, and this necessarily produced many irregularities in the final scheme of vectors. I have now gone over the data again, and by studying the balance of the various parts of the system have brought out the revised scheme herewith presented. Its well-balanced symmetry speaks strongly for its average accuracy, and it will be possible to draw out of it many important conclusions of fundamental value for theoretical meteorology. We may remark that none of the principles enunciated in the original report have undergone modification by this present review.

By comparing the vectors of figs. 6 and 7 of this paper with Tables 34-47 and Charts 15 and 16 of the Cloud Report, one may readily examine all the changes that have been adopted, and may also discover how closely these charts represent the mean system indicated by the original observations. Instead of carrying the discussion through on the mean cloud levels where the observations were made, it is more convenient to select certain planes upon which the average vectors are established for further discussion.

It is necessary first to establish the normal mean annual vectors representing the eastward drift to which the observed vectors are to be referred, in order to decompose them and obtain the anticyclonic and the cyclonic vectors by themselves. These normal vectors are given in Table 4, which is an extract from Table 33, III, International Cloud Report. The

eastward velocities are also represented by fig. 8, Total eastward velocities in high and low areas, which shows that the low areas drift eastward more rapidly than the high areas at all levels above the stratus, where they have about the same velocity, and that they drift northward in the United States in the upper levels, at a somewhat higher velocity than in the low levels. It is important to bear in mind that the results of our observations pertain only to the central portions of the North American Continent, eastward of the Rocky Mountains, where the cyclonic storm tracks have on the average a northeastward direction toward the Gulf of St. Lawrence. On the Rocky Mountain slope they have a movement toward the south before recurving in the Mississippi Valley. Generally the eastward drift has a small northward or southward component varying in the different parts of the world, and it is not quite proper to draw general conclusions for the entire hemisphere from the motion of the atmosphere in one district. Furthermore, since the cyclonic areas have a special vortical progression of their own, it seems probable that the average velocities observed in the high areas represent the true motion of the total mass of circulating air more correctly than would the mean of the high and the low areas. The normal eastward and northward components have, therefore, been chosen a little in excess of those given by observation for the high areas, and they are placed in Table 8.

TABLE 8.—Normal component velocities on six selected planes.

Height.	Eastward velocity.	Northward velocity.	Height.	Eastward velocity.	Northward velocity.
<i>Meters.</i>	<i>m. p. s.</i>	<i>m. p. s.</i>	<i>Miles.</i>	<i>m. p. h.</i>	<i>m. p. h.</i>
10,000	36	— 2	6.21	80	— 4
7,500	34	— 2	4.66	76	— 4
5,000	26	— 1.5	3.11	58	— 3
3,000	20	— 1	1.86	45	— 2
1,000	8	— 1	0.62	17	— 2
Surface	4	— 0.5	Surface	9	— 1

Two points may be noted in passing: (1) The eastward drift seems to be stratified into a series of steps by a decided change of the eastward velocity, and it appears that some form of stratus cloud is to be found at the bottom, and some form of cumulus cloud at the top, of each distinct stratum of flowing air. This indicates that at the surface of discontinuity between moving strata, the stratus type of cloud forms by a process of cooling through mixture from adjacent layers of air at different temperatures, which is in accord with general theory. It also shows that the cumulus clouds form by vertical convection and dynamic cooling within a stratum having about the same uniform velocity of motion throughout its mass and this is also theoretically correct. (2) The components of average total motion do not show that the atmosphere drifts northward in the higher levels and at the surface, but southward in the lower middle levels, somewhat elevated from the ground, as was claimed should be the case by Professor Ferrel in his canal theory of the general circulation of the atmosphere. I will return to this topic and consider it at length, but the fact here indicated is that the observations do not sustain that part of the general canal theory. It is becoming clearly demonstrated to students that the circulation of the air is a more complicated problem than the early meteorologists assumed, and in consequence it will be necessary to study in detail the stream lines over the several continents and oceans, find out their local

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FIG. 6.—Adjusted mean vectors of direction and velocity of motion in high areas. 10,000 meters = 6.21 miles. Scale of distances, 1 cm. = 500 kilometers; velocities, 1 mm. = 2 meters per second, 1 meter per second = 2.24 miles per hour.

FIG. 7.—Adjusted mean vectors of direction and velocity of motion in low areas. 10,000 meters = 6.21 miles. Scale of distances, 1 cm. = 500 kilometers; velocities, 1 mm. = 2 meters per second. 1 meter per second = 2.24 miles per hour.

characteristics, and after that try to combine them in a large comprehensive scheme.

DESCRIPTION OF THE CIRCULATION OVER HIGH AND LOW AREAS.

Figs. 6 and 7 represent the adjusted mean vectors of direction and velocity of motion in high and low areas, as derived from the Weather Bureau observations of 1896-97. They are based upon about 6,000 theodolite observations made at

ble without them. For the student of theoretical meteorology they are indispensable, because they correct the impressions which may be given by a contemplation of the figs. 6 and 7, or by reflecting upon the analytical formulae.

DISCUSSION OF THE VECTORS IN HIGH AREAS.

The area about the center of circulation was subdivided into twenty small parts, numbered as already described in a previous paper; the upper left-hand plans of figs. 6 and 7 show them again for convenience of reference. Through the center of each of the three concentric groups a circle is drawn in dotted lines, and these are marked I, II, III, their distance from the center being 250, 750, 1,250 kilometers, respectively. The adopted heights of the planes of motion in meters and miles are written on each level, also the normal velocity vector in meters per second (m. p. s.), and miles per hour (m. p. h.). The scale of distances is 1 cm. = 500 kilometers, and the scale of velocities is 1 mm. = 2 meters per second; the latter can be reduced to miles per hour by multiplying with the factor 2.24. The left-hand plans contain the total vector as observed in the atmosphere; the right-hand plans give the component vector, which, combined with the normal vector, produces the observed vector, using the rule of the parallelogram of vectors. Each vector has been carefully constructed and deserves considerable confidence. The smoothly balanced configuration in each level and the gradual change which occurs in passing from one level to another show that this represents a natural and easy form of flow for the atmosphere, so that the motion will occur without sharp changes. The figures speak plainly for themselves, and only a few words are required regarding the distinguishing features. In the high areas the total flow diminishes in strength from 10,000 meters to the surface; it has a slight curvature northward over the center in the highest level, but this concavity of the curves gradually increases till in the lower levels and at the surface the sinuous lines are converted into anticyclonic gyrations. The vectors north of the center are longer than those south of it from the top to the bottom. There is, however, a strong eastward drift in all levels, inward on the west side and outward on the east side, which is never overcome.

Passing now to the anticyclonic component vectors, it is noted that there is a remarkable symmetry in the configuration from the highest level to the lowest, taken as a whole. There are, however, two special features to be observed: (1) In the central areas, I, the flow is inward on the highest level, more from the north, however, than from the south; it is tangential on the middle level; and it is outward in the lowest level. This indicates a type of true vortex motion, which prevails at the center of anticyclones, and by it the air is drawn in at the top and discharged at the bottom of the vortex tube. (2) On the middle areas, II, the flow is nearly tangential throughout the entire series of strata, but on the outer areas, III, the vectors are pointed slightly outward from the top to the bottom, though more strongly on the east side than on the west side. There is, furthermore, the special feature that at the south or southwest side of the anticyclonic area, near the place marked A, a distinct discontinuity occurs in the vectors, by which on the west side an inflow from the south takes place, and on the east side an outflow from the north is indicated. I interpret these two facts together to mean that in the southeast quadrant there is a tendency for a heavy stream of the general circulation from the northwest to divide, so that a large portion moves to the south side of the adjacent cyclonic area and a small portion curls westward about the center of the high area. Also, on the west side of the high area a stream from the south divides, part flowing over the north of the high area and another part curling about the north side of the center of the adjacent low area. Fig. 9, Curling of the northward and southward streams about the centers of high and low areas,

FIG. 8.—Total eastward velocities in high and low areas.

Washington, D. C., and about 25,000 nephoscope observations made at 15 stations distributed quite uniformly over the territory east of the Rocky Mountains. They give only a mean or average scheme of the circulation and are necessarily somewhat idealized, as regards the movements of the air in individual configurations, since they include all the anticyclones and cyclones of the cloud year, many of which were only imperfectly developed, and could not have agreed with the best types that might have been selected. In order that no false impressions should remain with students concerning the actual circulation of the atmosphere, because of this construction of a well-balanced type, I compiled for the International Cloud Report a series of composite charts, Nos. 20 to 35, inclusive, which show the actual stream lines in high and low areas over the several areas of the United States, both for summer and winter. These charts are not only interesting, but they are very valuable, because they give the normal flow of the air when the anticyclonic and cyclonic centers are located in different parts of the country. They ought to be studied carefully by every forecaster, and the general knowledge given by the charts should be kept firmly in mind when considering the meaning of the individual daily weather maps, as they will guide the judgment to safer conclusions than would be possi-

FIG. 9.—Curling of the northward and southward streams about the centers of high and low areas.

gives an idea of this process, especially in the strato-cumulus level, or at about 3,000 meters elevation. The heavy broken line represents the resulting sinuous eastward flow at that level. In the flow of fluids a wave motion, when the velocity exceeds a given amount, collapses and reappears in the form of whirls of discontinuous surfaces along the sides. Something of this sort is apparently operating in this connection.

We observe that in the 3,000-meter level the anticyclonic vectors are stronger than in the levels above or below, the diminution toward the surface being greater than toward the higher levels. The superposition of the component gyration upon the eastward drift is distinct and even vigorous at 10,000 meters, and hence it is inferred that the disturbance of the atmosphere in high areas extends to at least 6 or 8 miles, though only as a small deflection of the eastward drift in the upper strata.

TABLE 9.—Rectangular and cylindrical coordinates in high areas.

Direction from center.	Area number.	10,000 meters.				7,500 meters.				Distance from center.
		u_1	v_1	u_2	v_2	u_1	v_1	u_2	v_2	
S	1	-2	+29	-2	-6	+2	+24	+2	-10	I. 250 km.
E	2	+6	+31	-4	-6	+8	+30	-4	-8	
N	3	+6	+40	-6	-5	+6	+36	-6	-2	
W	4	+4	+39	-3	+5	-4	+32	+2	-4	
S	5	+8	+28	+8	-8	0	+24	0	-10	
SE	6	+10	+30	+2	-9	+8	+30	+3	-9	II. 750 km.
E	7	+7	+40	+4	-7	+8	+36	+2	-8	
NE	8	+6	+43	+2	-7	+8	+38	-4	-8	
N	9	+2	+44	+2	-8	+4	+40	-4	-6	
NW	10	-4	+40	-2	-6	-8	+38	+4	-8	
W	11	-4	+36	-1	-4	-10	+30	+4	-10	III. 1,250 km.
SW	12	-5	+28	0	-7	-8	+30	-4	-8	
S	13	+8	+30	+8	-6	+4	+24	+4	-10	
SE	14	+9	+28	+2	-9	+6	+28	0	-9	
E	15	+9	+40	+4	-9	+10	+38	+4	-10	
NE	16	+10	+42	-4	-10	+9	+40	-4	-8	
N	17	-2	+44	+2	-8	+3	+42	-3	-8	
NW	18	-8	+40	+4	-8	-6	+40	+1	-8	
W	19	-8	+32	+4	-8	-9	+30	+4	-9	
SW	20	-8	+32	-4	-6	-8	+28	-6	-8	

DISCUSSION OF VECTORS IN LOW AREAS.

The vectors in the low areas should in general be a little longer than those in the high areas. In nature the highs cover a larger territory than do the lows, but as the amount of air which streams through each of them is probably about the same, it would require a greater velocity in the lows to produce an equal discharge through them. The vectors flow southward relatively to the center, and they are larger on the southern side than on the northern. The connection of the streams between the high and low areas is shown by the smooth flow of the two sets of vectors on their eastern and western sides, respectively. The stream lines are convex upward, and the curvature increases from the 10,000-meter level to the surface. In the 1,000-meter level the gyrotory movement nearly supersedes the sinuous or wave-like flow, but the vectors on the north side are not entirely reversed to the westward.

TABLE 10.—Rectangular and cylindrical coordinates in low areas.

Direction from center.	Area number.	10,000 meters.				7,500 meters.				Distance from center.
		u_1	v_1	u_2	v_2	u_1	v_1	u_2	v_2	
S	1	-10	+46	-10	+10	-6	+46	-6	+12	I. 250 km.
E	2	-12	+34	-2	+12	-12	+32	-2	+12	
N	3	-4	+32	+4	+4	-6	+26	+6	+8	
W	4	-6	+40	-6	-4	+4	+44	-10	+4	
S	5	-8	+44	-8	+8	-10	+44	-10	+10	
SE	6	-12	+42	-5	+14	-12	+34	-6	+10	II. 750 km.
E	7	-10	+32	-4	+10	-10	+30	-4	+10	
NE	8	-4	+24	-4	+12	-6	+26	-4	+9	
N	9	-6	+32	+4	+6	-4	+28	+4	+6	
NW	10	-3	+30	+6	+3	+4	+26	+4	+8	
W	11	+6	+44	-6	+8	+8	+42	-8	+8	III. 1,250 km.
SW	12	+2	+46	-6	+8	0	+46	-7	+10	
S	13	-4	+44	-4	+8	-4	+42	-4	+8	
SE	14	-8	+40	-4	+8	-8	+38	-4	+8	
E	15	-6	+36	0	+6	-8	+30	-4	+8	
NE	16	-4	+28	-4	+8	-8	+28	+2	+10	
N	17	-4	+30	+4	+6	-4	+30	+4	+4	
NW	18	-4	+30	+6	+4	-2	+28	+6	+2	
W	19	+6	+42	-6	+6	+6	+40	-6	+6	
SW	20	+4	+42	-4	+6	+4	+40	-2	+7	

TABLE 9.—Rectangular and cylindrical coordinates in high areas—Cont'd.

Direction from center.	Area number.	5,000 meters.				3,000 meters.				Distance from center.
		u_1	v_1	u_2	v_2	u_1	v_1	u_2	v_2	
S	1	+ 2	+18	+ 2	- 8	+ 4	+12	+ 4	- 8	I. 250 km.
E	2	+ 8	+22	- 2	- 8	+ 8	+16	- 4	- 8	
N	3	+ 4	+34	- 4	- 8	- 2	+26	+ 2	- 6	
W	4	- 8	+28	- 2	- 8	- 8	+16	+ 4	- 8	
S	5	+ 4	+16	+ 4	-10	+ 6	+14	+ 6	- 6	II. 750 km.
SE	6	+ 8	+20	+ 4	- 8	+12	+14	+ 4	-12	
E	7	+ 8	+28	+ 2	- 8	+10	+20	0	-10	
NE	8	+ 6	+32	0	- 8	+ 6	+28	+ 2	-10	
N	9	+ 3	+34	- 3	- 8	- 2	+28	+ 2	- 8	III. 1,250 km.
NW	10	- 6	+30	+ 1	- 7	- 8	+24	+ 2	-10	
W	11	- 8	+24	+ 2	- 8	- 8	+24	- 4	- 8	
SW	12	- 6	+20	0	- 8	- 6	+14	- 4	- 8	
S	13	+ 4	+16	+ 4	-10	+ 8	+10	+ 8	-10	
SE	14	+ 8	+22	+ 4	- 8	+10	+16	+ 5	-11	
E	15	+ 8	+30	+ 4	- 8	+10	+26	+ 6	-10	
NE	16	+ 7	+32	0	-10	+10	+28	- 2	-12	
N	17	0	+36	0	-10	+ 2	+30	- 2	-10	
NW	18	- 9	+32	+ 1	-11	-12	+24	+ 4	-12	
W	19	-10	+24	+ 2	-10	-10	+24	- 4	-10	
SW	20	- 8	+22	- 4	- 8	- 8	+14	- 4	-10	

Direction from center.	Area number.	1,000 meters.				Surface.				Distance from center.
		u_1	v_1	u_2	v_2	u_1	v_1	u_2	v_2	
S	1	+ 4	+ 2	+ 4	- 6	+ 3	0	+ 3	- 4	I. 250 km.
E	2	+ 6	+12	+ 4	- 6	+ 3	+ 7	+ 3	- 3	
N	3	- 6	+14	+ 6	- 6	- 4	+ 7	+ 4	- 3	
W	4	- 8	+ 6	+ 2	- 8	- 5	+ 2	+ 2	- 5	
S	5	+ 8	+ 4	+ 8	- 4	+ 3	+ 1	+ 3	- 3	II. 750 km.
SE	6	+10	+ 6	+ 6	- 8	+ 4	+ 2	+ 4	- 4	
E	7	+ 8	+10	+ 2	- 8	+ 6	+ 8	+ 4	- 6	
NE	8	+ 6	+16	+ 6	- 8	+ 4	+10	+ 2	- 7	
N	9	- 4	+16	+ 4	- 8	- 3	+10	+ 3	- 6	III. 1,250 km.
NW	10	- 8	+10	+ 1	- 9	- 5	+ 8	0	- 6	
W	11	-10	+ 8	0	-10	- 6	+ 2	+ 2	- 6	
SW	12	- 8	+ 2	- 2	-10	- 3	0	+ 2	- 5	
S	13	+ 8	0	+ 8	- 8	+ 4	- 1	+ 4	- 5	
SE	14	+10	+ 4	+ 6	- 9	+ 7	+ 5	+ 6	- 4	
E	15	+ 8	+14	+ 6	- 8	+ 6	+ 9	+ 5	- 6	
NE	16	+ 8	+16	+ 2	-11	+ 4	+10	+ 2	- 7	
N	17	- 4	+18	+ 4	-10	- 4	+ 8	+ 4	- 4	
NW	18	-10	+12	+ 4	-10	- 6	+ 8	+ 1	- 7	
W	19	-10	+10	- 2	-10	- 7	+ 4	0	- 7	
SW	20	-10	+ 4	- 4	-10	- 5	- 1	- 2	- 5	

+ u_1 = southward.
+ v_1 = eastward.

+ u_2 = radial outward.
+ v_2 = tangential counter clockwise.

The cyclonic components are very symmetrically formed throughout the entire stratum of air that has been examined. They have the following characteristic, namely, that from the surface to the 10,000-meter level the vectors have an inflow toward the center, except in a few subareas marked with the letter A. It is noted that from the 10,000-meter level to the 1,000-meter level, near the place A, the vectors are almost exactly opposed to each other in direction, those on the east side flowing outward and those on the west side flowing inward. This divergence of direction indicates that a stream flows from

TABLE 10.—Rectangular and cylindrical coordinates in low areas—Cont'd.

Direction from center.	Area number.	5,000 meters.				3,000 meters.				Distance from center.
		u_1	v_1	u_2	v_2	u_1	v_1	u_2	v_2	
S	1	-10	+42	-10	+16	-10	+40	-10	+20	I. 250 km.
E	2	-16	+22	- 4	+16	-18	+16	- 4	+18	
N	3	- 2	+16	+ 2	+10	- 8	+12	+ 8	+ 6	
W	4	+14	+28	- 6	+14	+16	+28	- 8	+16	
S	5	- 4	+36	- 4	+10	-10	+32	-10	+12	II. 750 km.
SE	6	-14	+32	- 5	+14	-12	+26	- 6	+12	
E	7	-12	+24	- 2	+12	-14	+24	- 4	+14	
NE	8	-10	+20	+ 4	+12	-12	+12	+ 4	+14	
N	9	-10	+18	+ 6	+10	-12	+14	+ 6	+12	III. 1,250 km.
NW	10	+12	+20	- 6	+12	+12	+18	- 7	+10	
W	11	+12	+30	- 4	+12	+16	+22	- 2	+16	
SW	12	+ 4	+38	- 4	+12	+12	+32	0	+18	
S	13	- 4	+32	- 4	+ 6	- 6	+28	- 6	+ 8	
SE	14	- 6	+30	- 4	+ 6	- 8	+26	- 2	+10	
E	15	- 8	+22	- 4	+ 8	-10	+18	- 2	+10	
NE	16	- 6	+22	+ 2	+ 7	-10	+14	+ 4	+10	
N	17	- 8	+20	+ 6	+ 8	-12	+14	+12	+ 6	
NW	18	+ 6	+22	- 2	+ 7	+10	+18	- 6	+ 8	
W	19	+ 8	+28	- 2	+ 8	+12	+24	- 4	+12	
SW	20	+ 4	+34	- 4	+ 8	+ 4	+28	- 4	+ 8	

Direction from center.	Area number.	1,000 meters.				Surface.				Distance from center.
		u_1	v_1	u_2	v_2	u_1	v_1	u_2	v_2	
S	1	- 6	+24	- 6	+16	- 4	+10	- 4	+ 6	I. 270 km.
E	2	- 8	+ 4	- 4	+ 8	- 6	0	- 4	+ 6	
N	3	+10	+ 4	-10	+ 4	+ 4	- 2	- 4	+ 6	
W	4	+10	+12	- 4	+10	+ 8	+ 8	- 4	+ 8	
S	5	- 4	+20	- 4	+12	- 4	+10	- 4	+ 5	II. 750 km.
SE	6	-10	+12	- 4	+10	- 6	+ 6	- 2	+ 6	
E	7	-10	+ 6	- 2	+10	- 6	+ 2	- 2	+ 6	
NE	8	-10	+ 8	+ 8	+ 6	- 4	- 2	- 2	+ 4	
N	9	+10	+ 4	-10	+ 4	+ 4	- 2	- 6	+ 4	III. 1,250 km.
NW	10	+12	+ 4	- 6	+10	+ 6	+ 2	- 4	+ 5	
W	11	+ 8	+14	- 6	+ 8	+ 6	+ 8	- 4	+ 6	
SW	12	+ 8	+20	- 4	+14	+ 4	+10	- 2	+ 8	
S	13	- 4	+18	- 4	+10	- 4	+ 8	- 4	+ 4	
SE	14	-10	+10	- 2	+10	- 6	+ 4	- 4	+ 4	
E	15	- 8	+ 8	0	+ 8	- 6	+ 4	0	+ 6	
NE	16	- 8	+10	+ 8	+ 4	- 4	- 2	- 2	+ 4	
N	17	+10	+ 6	-10	+ 2	+ 4	- 2	- 6	+ 4	
NW	18	+10	+ 8	- 9	+ 4	+ 6	0	- 4	+ 6	
W	19	+ 8	+14	- 6	+ 8	+ 6	+ 8	- 4	+ 6	
SW	20	+ 6	+14	0	+ 8	+ 3	+ 8	- 2	+ 5	

+ u_1 = southward.
+ v_1 = eastward.

+ u_2 = radial outward.
+ v_2 = tangential counter clockwise.

the north to the south on the west of the low area, and that an independent stream flows northward on the east side of the low area, something in the manner suggested on fig. 9. The separate streams from the north and from the south coalesce on the south side of the center of the low area, as they do on the north side of the high area, but the two streams have an origin outside the areas of high and low pressure, respectively. Furthermore, it is noted that while in the high area the position of the point A is nearly stationary in all the strata mapped out, on the contrary it rotates nearly 90° from the east of

north at the surface to the north of west in the highest stratum. The stream of warm air from the south curls around toward the west as it ascends from the surface to the upper levels, making a quarter of a helical revolution in an ascending spiral. The length of the vectors is greatest in the 3,000-meter level, 2 miles above the ground, and the vectors become gradually shorter upward and downward, diminishing more rapidly toward the surface. This agrees with the system of vectors in high areas, and shows that the influence of the cyclone extends above the 10,000-meter level, where it still deflects considerably the eastward drift, though it is most vigorous in the 3,000-meter level. The length of the vectors increases gradually from the III-areas to the I-areas, and averages about twice as long in the latter as in the former. In the anticyclonic components the III-vectors are even longer than the I-vectors, and they do not have any agreement with the simple vortex law $\omega = \text{constant}$, where ω is the radial distance from the axis of rotation, and ω the angular velocity.

In the cyclonic components the I-vectors are longer than the III-vectors, but they fall short of exact conformity with the pure vortex theory. The entire flow suggests, therefore, the conflict of two counterflowing, horizontal streams which tend to produce vortical rotation, but in fact fail to reach this ideal, except possibly in highly developed cases of severe storms. There is no evidence that these motions are primarily due to vertical convective currents developed through the local heating or cooling of restricted areas near the center of the cyclonic and anticyclonic areas, respectively. It is evidently desirable to avoid extreme statements in this connection, because a study of the motions of the atmosphere shows that nearly every possible type of motion from the counterflow of opposing horizontal streams to the pure vortex due to an ascending helix may occur, and yet the present compilation indicates that the former is the average type to which the stream lines conform in the extra-tropical circulation of the United States.

THE NUMERICAL VALUES OF THE VECTORS.

In order to bring out these facts a little more clearly, the vectors of fig. 6 have been translated into the numerical values of Table 9, Rectangular and cylindrical coordinates in high areas; and those of fig. 7 into the numbers of Table 10, Rectangular and cylindrical coordinates into low areas. These tables need no further explanation in this connection, after what has been already stated.

Table 11, Mean components on the I, II, III circles in meters per second and in miles per hour, is derived from the anticyclonic components of Table 9, and the cyclonic components of Table 10, by taking the arithmetical mean of the I-areas (1-4), the II-areas (5-12), and the III-areas (13-20). These means give the average value of the motion, though we, of course, depart from the perfectly natural condition by the summation. Thus in the anticyclonic areas for the radial component u_2 there is an inflow at the top of I-areas, and an outflow at the bottom; and a gentle outflow in the II-areas and III-areas from the top to the bottom. Also compare fig. 10, where the results of Table 11 are plotted. The tangential component v_2 is stronger throughout the middle strata than in those which are higher or lower, but it is much more vigorous in the III-areas than in the I-areas especially at the 3,000-meter level. In the cyclonic areas the radial component u_2 increases generally from the III-area to the I-area. There is a little irregularity in the changes of this component probably due to imperfections in my vector system. The tangential component v_2 increases rapidly from the III-areas to the I-areas, and remarkably so at the 3,000-meter level.

It has been taught in the common expositions of the canal theory of the general circulation that there exists in middle latitudes a strong northward component in the upper strata, a strong southward component in the surface and lower strata,

and a powerful eastward component in all strata, increasing from the ground upward. It can be seen by inspecting figs.

TABLE 11.—Mean components on I, II, III circles.

ANTICYCLONIC COMPONENTS.						
Distance from center.	I. 250 kilometers.		II. 750 kilometers.		III. 1,250 kilometers.	
Meters per second.	u_2	v_2	u_2	v_2	u_2	v_2
$H=10,000$	— 3.8	— 3.0	+ 1.9	— 7.0	+ 2.0	— 8.0
7,500	— 1.5	— 6.0	+ 0.1	— 8.4	0.0	— 8.8
5,000	— 1.5	— 8.0	+ 1.3	— 8.1	+ 1.4	— 9.4
3,000	+ 1.5	— 7.5	+ 1.0	— 9.0	+ 1.4	— 10.6
1,000	+ 4.0	— 6.5	+ 3.1	— 8.1	+ 3.0	— 9.5
0	+ 3.0	— 3.8	+ 2.5	— 5.4	+ 2.5	— 5.6
CYCLONIC COMPONENTS.						
$H=10,000$	— 3.5	+ 5.5	— 2.9	+ 8.6	— 1.5	+ 6.5
7,500	— 3.0	+ 9.0	— 3.9	+ 8.9	— 1.0	+ 6.6
5,000	— 4.5	+ 14.0	— 1.9	+ 11.8	— 1.5	+ 7.3
3,000	— 3.5	+ 15.0	— 2.4	+ 13.5	— 1.0	+ 9.0
1,000	— 6.0	+ 9.5	— 3.5	+ 9.3	— 2.9	+ 6.8
0	— 4.0	+ 6.5	— 3.3	+ 5.5	— 3.3	+ 4.9
ANTICYCLONIC COMPONENTS.						
Distance from center.	I. 155 miles.		II. 466 miles.		III. 777 miles.	
Miles per hour.	u_2	v_2	u_2	v_2	u_2	v_2
$H=10,000$	— 8.5	— 8.7	+ 4.3	— 15.7	+ 4.5	— 17.9
7,500	— 3.4	— 13.4	+ 0.2	— 18.8	0.0	— 19.7
5,000	— 3.4	— 17.9	+ 2.9	— 18.1	+ 3.1	— 21.0
3,000	+ 3.4	— 16.8	+ 2.2	— 20.1	+ 3.1	— 23.7
1,000	+ 8.9	— 14.5	+ 6.9	— 18.1	+ 6.7	— 21.3
0	+ 6.7	— 8.5	+ 5.6	— 12.1	+ 5.6	— 12.5
CYCLONIC COMPONENTS.						
$H=10,000$	— 7.8	+ 12.3	— 6.5	+ 19.2	— 3.4	+ 14.5
7,500	— 6.7	+ 20.1	— 8.7	+ 19.9	— 2.2	+ 14.8
5,000	— 10.1	+ 31.3	— 4.3	+ 26.4	— 3.4	+ 16.3
3,000	— 7.8	+ 33.6	— 5.4	+ 30.2	— 2.2	+ 20.1
1,000	— 13.4	+ 32.4	— 7.8	+ 20.8	— 6.5	+ 15.2
0	— 8.9	+ 14.5	— 7.4	+ 12.3	— 7.4	+ 11.0

TABLE 12.—Northward and southward velocities in selected areas.

Height of the stratum.	Northward.		Southward.	
	I. 16, 8, 2, 7, 15, 6, 14. II. 18, 10, 4, 11, 19, 12, 20.		H. 16, 8, 2, 7, 15, 6, 14. L. 18, 10, 4, 11, 19, 12, 20.	
	u_1	v_1	u_1	v_1
10,000	— 6.4	+ 34.5	+ 4.4	+ 37.7
7,500	— 8.4	+ 31.9	+ 5.8	+ 36.2
5,000	— 9.1	+ 25.2	+ 8.1	+ 27.6
3,000	— 10.3	+ 19.7	+ 10.6	+ 22.7
1,000	— 9.2	+ 7.9	+ 8.4	+ 11.7
Surface	— 5.2	+ 2.6	+ 5.3	+ 6.9

Compare Table 124, International Cloud Report, p. 606.

FIG. 10.—Radial and tangential components in anticyclonic and cyclonic areas. From Table 11.

6 and 7 that while there is everywhere a general eastward drift, there are certain subareas over which especially a northward component prevails, and others over which there is a southward component. In order to find the maximum meridional components it is expedient to select the following areas for the northward component: Low (16, 8, 2, 7, 15, 6, 14) and High (18, 10, 11, 19, 12, 20), and for the southward component High (16, 8, 2, 7, 15, 6, 14) and Low (18, 10, 4, 11, 19, 12, 20). The values of u_1 , v_1 are taken for these areas from Tables 9 and 10, and the mean of them is given in Table 12, Northward and southward velocities in selected areas. It can be seen at once that the general canal theory is by no means supported by the observations. The fact seems to be that between the high and low centers, west of the high and east of the low, there is a northward current in all levels, strongest at about the 3,000-meter level, while east of the high and west of the low there is a southward current also strongest in the same level. The interchange of air between the pole and the Tropics appears, therefore, to be brought about by alternate currents in middle latitudes flowing past each other on the same levels, and not over each other at entirely different levels, as the canal theory requires. The thermal equilibrium of the air is, therefore, restored through the anticyclonic and cyclonic mechanism, and not by the overflowing currents from the Tropics to the poles and underflowing currents from the poles to the Tropics, as commonly taught. This profoundly modifies the canal theory of the general circulation of the atmosphere and introduces us to a new point of view. The discussion of the theories of the circulation of the air as explained by Ferrel, Oberbeck, and other meteorologists must be taken up next in order, and their views contrasted with the results of our observations.

IV.—REVIEW OF FERREL'S AND OBERBECK'S THEORIES OF THE LOCAL AND THE GENERAL CIRCULATIONS.¹

GENERAL COMPARISON OF FERREL'S AND OBERBECK'S THEORIES.

In order to discuss the theories which have been proposed to account for the circulation of the atmosphere in cyclones and anticyclones, and in general over an hemisphere of the earth, it will be convenient to confine our attention to the views propounded by Ferrel and Oberbeck because their treatment is quite complete, and also because they represent a large number of writers who agree with them more or less perfectly. There is another theory of quite a different type which can be taken up profitably after some critical remarks have been made on the validity of these earlier views. In their treatment of the general circulation of the atmosphere both Ferrel and Oberbeck adopt the "canal theory" of the circulation, and work out their solutions along that line. Oberbeck places his origin of coordinates at the center of the rotating earth, develops the equations of motion, and transforms to the surface when they are employed in the evaluation of the resulting velocities. He also deduces the terms in the pressure due to the absolute motion of the earth, and to the relative motions of the atmosphere. Ferrel places his origin of coordinates at the surface of the earth, transforms his equations to include a temperature term through the variations of the density, and discusses the meaning of the equations under special limitations, with illustrations from the observed motions of the atmosphere. It may be remarked that von Helmholtz introduces the temperature terms into the equations of motion, not through the density, but through the pressure, by using the equation of elasticity, $p v = R T$. This procedure is probably a better method of solution. There is not much difference in the results as derived from the analysis by these authors, but there is serious difficulty in making them agree with the modern observations of the motions of the atmosphere in the higher strata, as determined by the international cloud work.

In their treatment of the cyclone Ferrel and Oberbeck diverge radically from each other, though they start with the same physical principle, namely, a local overheated mass of air which in rising through its own buoyancy produces the cyclonic circulation. Ferrel assumes a fixed cylindrical boundary about his cyclone, and considers a warm-center cyclone (circulation anticlockwise), surrounded by a pericyclonic ring (circulation clockwise), in the Northern Hemisphere, the two portions being separated by a surface where the gyratory velocity vanishes. By maintaining a cold mass of air in the center instead of a warm mass the circulation is reversed, and a cold-center cyclone is developed. Oberbeck does not assume any external boundary to the circulating mass of air, but in the central region, bounded by a cylindrical surface, there is a vertical component, while outside of it there is no such vertical ascent of the air. At this boundary there is a discontinuity in the vertical velocity, and at the same distance from the center the gyratory velocity about the axis is a maximum; this falls away to zero at the center and also at some indefinite distance in the outer region. It is essential to the existence of these two theories, although they differ so radically from each other, to establish the fact that such local centers of heated air in the warm-center cyclones do occur in nature, for without them these two theories entirely fail of applicability to our meteorology. They are both possible forms of vortex motion, but it is necessary to show that the antecedent physical conditions prevail, before they can be accepted as explanations of the observed cyclonic motions.

Both of these authors have experienced much difficulty in accounting for the anticyclones. Ferrel explained that the interference of two of his pericyclonic rings would heap up the air and produce an area of high pressure with a clockwise outflow, but this theory is so far from being in conformity with the facts, that it is now, by general consent of meteorologists, considered to be of only historical value. Oberbeck sought, by simply reversing the sign of the vertical component of velocity, to invert his cyclone into an anticyclone. He met with a stumbling-block in the mathematical analysis, but was relieved of this by Pockels, who correctly evaluated the constant of integration. No attempt was made to show that the resulting stream lines conform to the motions of the air in high areas of pressure. Indeed, since the modern observations have given us more correct lines of flow, it is quite certain that the anticyclone can not be explained in this way.

THE SUPPLY OF LOCAL CENTERS OF HEAT.

It is evident, therefore, that the first practical question to decide is whether such local masses of air exist, heated in the under strata and more or less cylindrical in form, as will produce either of the above forms of cyclone. Meteorologists have generally supposed that this is the case, and they have usually attributed the source of the vertical convection to the latent heat of condensation. Dr. J. Hann, in 1890, and again in his *Lehrbuch der Meteorologie*, has shown in great detail the inadequacy of this source of heat to produce cyclones, and he has indicated that the source of cyclonic action consists rather in horizontal convection currents. As this agrees with the view which I have already advocated, since it seems to me to be in conformity with the observations, I will therefore make a résumé of my remarks on this topic in the International Cloud Report. It will be a great gain if meteorologists can be persuaded to reject the old condensation theory, which has an apparent but really illusory plausibility, in favor of the really efficient source of dynamic action contained in the long, horizontal currents which flow between the Tropics and the polar regions in the middle strata of the atmosphere, as illustrated in the preceding Paper III.

There is, in fact, a fundamental difficulty in accounting for the local supply of heat which is assumed to set the vertical convection in operation. Ferrel himself doubted the efficiency of the latent heat of condensation, for he says in his *Meteorological Researches*, Appendix No. 10, United States Coast and Geodetic Survey, Part II, page 201: "The condensation of aqueous vapor, therefore, plays an important part in cyclonic disturbances, but is by no means a primary or a principle cause of cyclones." Professor Loomis asserted in *Silliman's Journal*, July, 1877: "Rainfall is not essential to the formation of areas of low barometer, and is not the principle cause of their formation or of their progressive motion." Indeed, a reasonable familiarity with the United States weather maps proves conclusively that there are many deep, fully-developed storms which form near the north Pacific coast and advance to the Gulf of St. Lawrence without any precipitation worth mentioning. Also, cyclones form frequently in the southern Rocky Mountain districts and advance into the lower Mississippi Valley without any important rainfall; from that region onward in their course the precipitation and intensity of the storm often greatly increase, since the latent heat derived from the inflowing moist air of the Gulf of Mexico undoubtedly assists the vertical convection in the center of the cyclone. If the

¹ Reprinted from the Monthly Weather Review for April, 1902.
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horizontal currents which converge upon a cyclonic center are bearers of moisture, the vertical motion caused by the dynamic action condenses the aqueous vapor; but if such currents are dry, the cyclone advances unattended by precipitation. Hence, it follows that rainfall is a secondary phenomenon, and is not sufficient of itself to produce true cyclonic gyrations. There are, on the other hand, many cases of copious precipitation without any attendant low pressure. Thus, on the front of an advancing cold wave there is often a long band of rain area stretching from the Great Lakes to the Gulf of Mexico, but without cyclonic formation, the precipitation being in fact caused by the upward lift of a warm southerly current which overflows the wedge-shaped cold wave in its southward movement. This is a dynamic uplift by overflow, instead of by vortical gyration, and it is sufficient to cause condensation and precipitation by the mechanical action of an underflowing stratum of very low temperature. Furthermore, on one side of a mountain range, as the Alps, rainfall is observed to occur in the midst of the high pressure, while on the other side of the mountains the atmosphere is clear and the pressure is relatively low, thus reversing the required conditions. In the summer season, local thunderstorms are quite as likely to happen in the midst of an area of high pressure as in that of low pressure, but here the vertical convection distinctly exists and arises from a superheating of the lower strata. If buoyancy of the lighter air is the principal cause of the gyration of cyclones, then we should expect to find a similar rotatory motion developed in the formation of cumulus clouds and thunderstorms in hot summer weather, when the vertical component is evidently strong. But, on the contrary, while the ascension of the heated air is clearly visible in these clouds, there is usually no evidence of gyration of the cyclonic type. It has been found by Hann's mountain observations and by the Berlin balloon ascensions that the temperature of the central portions of the cyclone is colder than the temperature in the midst of the anticyclone at the same levels. Hence, if the relative density of the air column is the source of cyclonic gyration, we perceive that this fact is in direct contradiction to the requirements of the condensation theory, which demands that the central column of the cyclone shall be warmer than its surroundings.

Since the advocates of the condensation theory of cyclones usually regard the generation of the tropical hurricanes as the best example of that source of gyratory energy, it may be proper to state that the observed facts do not appear to sustain the theory. For (1) there is no evidence of a decided increase in the local temperature at the center of hurricanes. In this connection it is believed that the sudden rise in temperature in the Manila hurricane of October 20, 1882, was due to the direct radiation of the sun through the calm eye of the storm; (2) the winds are not sufficiently changed in direction at the feeble ring of high pressure to conform to the Ferrel pericyclone; they should be turned through at least 90° more in azimuth; (3) the conditions of heated, saturated air prevail in the Tropics throughout the year, but the hurricanes are produced at certain seasons only, and these are the times when the counter currents of the trades are most active at their northern and southern limits. Dr. Hann rejects the rain theory, and adopts the counter current theory for hurricanes: *Lehrbuch der Meteorologie*, pp. 563-566. It can be proved conclusively from observations that two counter currents flow together at the places where tornadoes are formed, where the tropical hurricanes are generated, and also where the cyclones of the middle latitudes are produced. These currents are especially active in the strata one or two miles above the ground, and this is probably the reason why they have not received due attention in constructing the theory of storms. It may be concluded that the local overheated central region does not exist in cyclones as the chief cause of their motion, and that the theories fail which depend upon it. There are,

however, other serious difficulties of a mathematical nature to which attention must be directed.

FERREL'S LOCAL CYCLONE.

On page 595, and following, of the International Cloud Report, the fundamental formulæ and assumptions, as employed by Professor Ferrel in his discussion of the local cyclone, are summarized, and an abstract for our purposes, in the notation already described in Paper II of this series, *MONTHLY WEATHER REVIEW*, February, 1902, p. 81, is given in the following lines:

Cylindrical equations of motion applicable to the local circulation in cyclones. See International Cloud Report, p. 502.

$$185. \quad \begin{aligned} -\frac{1}{\rho} \frac{\partial P}{\partial \varpi} &= \frac{du}{dt} - \left(2n \cos \theta + \frac{v}{\varpi}\right) v + kv, \\ -\frac{1}{\rho} \frac{\partial P}{\partial \varphi} &= \frac{dv}{dt} + \left(2n \cos \theta + \frac{v}{\varpi}\right) u + kv, \\ -\frac{1}{\rho} \frac{\partial P}{\partial z} &= \frac{dw}{dt} + g. \end{aligned}$$

Assumptions made in discussing these equations:

1. The temperature is a function of ϖ only, varies along the radius, but not with the altitude, and is symmetrical about the center.
2. The local cyclone is symmetrical about the axis of gyration, and is bounded by a cylindrical surface whose constant radius is ϖ_0 .
3. The friction is proportional to the relative velocity of two adjoining strata.
4. The assumed law of the variation of temperature along the radius is as follows, the isotherms being circles about the center:

$$t = A_0 + \frac{1}{2} (t_c - t_0) \cos \frac{\varpi}{\varpi_0} \pi.$$

5. In integrating for the law of the preservation of areas it is assumed that there is no friction between the air and the surface of the earth.

6. All forces depending upon the vertical velocity of the currents can be neglected, $w = 0$.

7. P_0 = the pressure for $h_0 = 0$.

The equations of motions become the following by applying these assumptions and transforming the pressure term:

$$397b. (1.) \quad -\frac{\partial \log P_0}{a \partial \varpi} = \frac{du}{dt} - \left(2n \cos \theta + \frac{v}{\varpi}\right) v + kv \\ - \frac{gh a}{\varpi_0 (1 + at)} \frac{1}{2} (t_c - t_0) \pi \sin \frac{\varpi}{\varpi_0} \pi;$$

$$(2.) \quad 0 = \frac{dv}{dt} + \left(2n \cos \theta + \frac{v}{\varpi}\right) u + kv;$$

where $a = \frac{1}{gl(1 + at)}$, by Table 64, 23; t_c = temperature at the center; and t_0 = temperature at the outer boundary of the cyclone.

As the result of the discussion of these two equations Ferrel deduces his cyclone which is represented in fig. 11. The corresponding cold-center cyclone is shown in fig. 12.

The first of the above assumptions regarding the distribution of the local temperature does not sufficiently conform to the data on the weather maps to be satisfactory, because the southeast section of a cyclone in the United States is usually much warmer than the northwest section. The symmetrical distribution of pressure about the center, where $-\frac{1}{\rho} \frac{\partial P}{\partial \varphi} = 0$,

is found in highly developed cyclones, and may be admitted in the analysis. The friction term is of minor importance with respect to the general theory of a cyclone which we are considering, and the vertical force derived from w may be neglected, though not the vertical velocity itself.

There are two entirely different methods of treating the second equation of motion,

$$397b. (2.) \quad \frac{dv}{dt} + \left(2n \cos \theta + \frac{v}{\omega}\right) u + kv = 0,$$

and this is the parting of the ways between (1) Ferrel's theory and (2) the German theory. The primary question to be kept in mind is, does the result of the observations conform exactly to either of these theories? This equation can be integrated by omitting the friction term kv and assigning an outer boundary; or it may be solved by a simple transformation, since two roots can be found, and the discussion of the group of general equations of motion carried forward with these. The former method is Ferrel's, and the latter is that of the German school, namely, Guldberg and Mohn, Sprung, Oberbeck, Pockels, and others.

FERREL'S SOLUTION.

Neglecting the friction term, the equation 397b (2) can be transformed, by substituting $u = \frac{\partial \omega}{\partial t}$ and multiplying by ω , into

$$2n \cos \theta \cdot \omega \frac{\partial \omega}{\partial t} + \omega \frac{dv}{dt} + v \frac{\partial \omega}{\partial t} = 0.$$

It should be noted that $\frac{dv}{dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial \omega} + v \frac{\partial v}{\partial \varphi}$.

Ferrel integrates as if $\frac{dv}{dt} = \frac{\partial v}{\partial t}$, omitting the two other terms; indeed, Ferrel was neglectful about the distinction between the total and the partial differentials in several portions of his work. The integration gives $\omega^2 \left(n \cos \theta + \frac{v}{\omega} \right) = c$, for each particle. Assigning an outer boundary ω_0 as the limit of integration, then, for the entire rotating mass, we deduce,

$$\omega^2 \left(n \cos \theta + \frac{v}{\omega} \right) = \frac{1}{2} \omega_0^2 n \cos \theta; \text{ and hence,} \\ v = \left(\frac{\omega_0^2}{2\omega^2} - 1 \right) \omega n \cos \theta,$$

where v = the tangential velocity at the distance ω from the axis of rotation. If $v = 0$, $R = 0.707 \omega_0$, where R is the radius of the circle at which the gyratory velocity reverses its direction. The locus of this R is indicated on fig. 11 for the warm-center cyclone and on fig. 12 for the cold-center cyclone; these two figures also show, in a general way, the circulation in this type of vortices. It will not be necessary to explain it further in this connection, but it is especially important to observe that Ferrel came to this vortex by the demands of his integration, and that he sought to uphold it by resorting to such physical sources of energy as seemed to be available. He had already applied an entirely similar process to his discussion of the circulation of the atmosphere of the earth over an entire hemisphere, but in that case it was, at least in part, justified by the fact that the air on the hemisphere continues to remain the same mass, so that integration between the pole and the plane of the equator was a proper procedure. Yet, in comparing this vortex with the circulation as displayed in figs. 6 and 7 of Paper III, we must consider the other objections besides the difficulty of accounting for local supply of heat in the central portions which is needed to keep the vortex in motion.

(1) Ferrel conceived the general cyclone on the hemisphere to be one with a cold center, since the poles are cold and the Tropics warm; and then the modification was made that the local cyclone is one with a warm center with the edges cooler than the middle portions. If a quantity of water be placed in a cylindrical vessel, and sawdust or some other material be scattered in it to show the lines of the circulation, and if this be rotated on a turntable, a form of motion can be produced quite similar to the one Ferrel proposed to explain the mechanism of storms. This circulation can be generated by any agency

which will make a vertical current in the center of the fluid, as a lamp on the lower side, or a paddle screw at the top. A lump of ice on the center of a rotating plane will give a circulation which is like that of the general cyclone over the hemisphere. Now this experiment is open to at least three objections of a very serious nature when it is attempted to apply the lines of the model to the processes in nature. It is not enough to show that there is an inflow at the bottom and an outflow at the top, in logarithmic spiral curves, to conclude that the analogue is satisfactory. Therein lies an assumption which in fact begs the entire question. The great difficulty is

FIG. 11.—Ferrel's circulation in warm-center cyclones.

FIG. 12.—Ferrel's circulation in cold-center cyclones.

that the circulation in the general cyclone consists of the same mass of air, which repeatedly passes through certain paths in consequence of the boundary conditions. The limited mass of fluid in the cylindrical vessel is also the same mass set in circulation, being bounded by the top and bottom surfaces and the curved sides, corresponding with the ground and top of the moving air and the plane of the equator. It has by no means been shown that the air concerned in the local cyclone consists of the same air moving over and over again in similar paths, and it is first necessary to do this in order to establish an analogue of that kind. The evidence from the cloud circulations proves that the cyclone is a form of circulation of the stationary type of configuration, through which fresh portions of the atmosphere continue to stream. If such is the case the analogue described above is inapplicable, and the deductions which have been so commonly drawn from it are quite incorrect. (2) There is no pericyclone discoverable in the records based upon many storms. Ferrel tried to show that the high-area pressures observed on the maps are the resultants of two or more overlapping pericyclones. But the detailed construction shown in Charts 15-35 of the International Cloud Report gives no support to this form of circulation. (3) Evidence of true cyclonic outflow at the top at some distance above the ground is probably entirely lacking. The cyclonic components of Table 10, paper III, prove that the radial velocity is inward from the ground to the top of the cyclone. It is not our purpose at this point to explain the principles of the circulation that actually exists, but simply to indicate that the Ferrel cyclone, though perfectly possible under certain conditions, is not the type which storms follow in their construction. It is certain that the supposed analogue between the local and the general cyclone is not sustained by the evidence, and if the observed movement of the atmosphere can be accounted for on other principles, in conformity with the observations, it will only

add to the force of the position here taken that the Ferrel local cyclone is merely one of many idealized cases. For these reasons we therefore are obliged to conclude that the Ferrel cyclone by no means conforms to the natural circulation, and need not be further considered. Indeed, Ferrel's teaching regarding the origin of cyclones and anticyclones should be eliminated from modern meteorology.

THE GERMAN SOLUTION.

If we make the abbreviation $\lambda = 2n \cos \theta$, retain the friction term, and make $\frac{dv}{dt} = \frac{\partial v}{\partial t}$, thus rejecting the two small terms, equation 397b (2) becomes:

$$422 \quad \frac{\partial v}{\partial t} + \frac{uv}{\omega} + \lambda u + kv = 0.$$

Taking $\frac{\partial v}{\partial t} = \frac{\partial v}{\partial \omega} \frac{\partial \omega}{\partial t} = u \frac{\partial v}{\partial \omega}$, we obtain

$$437 \quad u \frac{\partial v}{\partial \omega} + \frac{uv}{\omega} + \lambda u + kv = 0.$$

There are two solutions of this equation, as shown on pages 598 and 599 of the Cloud Report, namely:

First solution (inner).

Second solution (outer).

$$u = -\frac{c}{2} \omega.$$

$$u = -\frac{c}{\omega}.$$

$$v = +\frac{\lambda}{k-c} \cdot \frac{c}{2} \omega = -\frac{\lambda}{k-c} u.$$

$$v = +\frac{\lambda}{k} \cdot \frac{c}{\omega} = -\frac{\lambda}{k} u.$$

These can be expressed in two general laws:

(1.) Parabolic law.

(2.) Hyperbolic law.

$$\frac{u}{\omega} = -\frac{c}{2} = \text{constant}.$$

$$u\omega = -c = \text{constant}.$$

$$\frac{v}{\omega} = +\frac{\lambda}{k-c} \cdot \frac{c}{2} = \text{constant}.$$

$$v\omega = +\frac{\lambda}{k} c = \text{constant}.$$

These solutions are readily verified by substitution in the second equation of motion, 397b, and the two forms give rise, respectively, to parabolic surfaces on the inside of a certain circle, and to hyperbolic surfaces on the outside of it. Their discussion is given on page 509; an electrical analogue is explained on page 521, and they are further illustrated on pages 619 to 622 of the International Cloud Report. A diagram of the motion is shown in fig. 13 of the present paper. The result is that there is an outer region in which there is no vertical component, $w = 0$, and an inner region in which there is a vertical component which increases with the altitude, $w = cz$; see page 621, Cloud Report. These two regions are separated by a circle where the tangential component velocity, v , is a maximum; the velocity of rotation falls away to the center by the parabolic law, and also for an unlimited distance outward by the hyperbolic law. The inner region has the isobars separated from each other by distances conforming to the law, $d_1 = R^2 - \omega_1^2$, where R is the radius of the circle of maximum velocity, and ω_1 the radii of the successive isobars; on the outside the distances between isobars are determined by $d_e = 2 R^2 \log \frac{\omega}{R}$; fig. 13 shows these relative distances and velocities.

Recalling the circulation depicted in Paper III, we are induced to make the following remarks:

The theory common to the German school of meteorologists is founded upon the assumption of a vertical central current, like the electric current in a wire, which generates the cyclonic circulation in the inner and the outer parts. Now, there are a series of difficulties and objections to this view, when it is attempted to apply it to the observations of the actual motions of the atmosphere, fully as serious as those which have been urged against Professor Ferrel's theory. (1) There is no sufficient evidence that the vertical current is of definite local

origin and powerful enough to influence the enormous cyclonic disturbances extending horizontally to 1,000 miles in radius. These storms are very shallow compared with their width, say 3 or 4 to 1,000 at the greatest depth. An upward central current in a small inner region of 200 to 300 miles radius, even if locally produced, could hardly cause the disturbances observed on the weather maps. The chief difficulty has been to show that there is any sufficient cause for the existence of such a current, and the reasons already urged against that view hold

FIG. 13.—Oberbeck's circulation in warm-center cyclones.

here also, namely, that the disturbing isotherms are not circles, but their gradients lie athwart the cyclone, generally from SW to NE; that cyclones exist without precipitation; that rainfall does not necessarily produce cyclonic action; and that countercurrents from two specific directions, as NW and S, feed into the cyclone, which is not sustained from a supply equally distributed around a center. (2) The adoption of the inner and outer parts of the cyclone was due to the supposed necessity of avoiding infinitely great velocities at the center, if $u = -\frac{c}{\omega}$ and $v = +\frac{\lambda}{k} \frac{c}{\omega}$, as would occur for small values of ω .

It will, of course, be necessary to show how this can be done by some other solution. Even if that is accomplished we find still other practical difficulties in the German solution having an inner and an outer part. This requires a maximum rotational velocity v at the boundary $\omega = R$. But our observations give no support to this position any more than to Ferrel's theory that $v = 0$ at the boundary of the inner and the outer parts. A careful examination of our wind velocities shows that they increase steadily from the outer boundary toward the center, when a surface of discontinuity surrounding a calm center suddenly terminates the radial and tangential velocities. The common occurrence of the central calm in hurricanes is sufficient proof of this point. Furthermore, an examination of the cyclonic components (v_r, v_t), Table 10 and fig. 10, shows that the tangential velocity v increases from the outside toward the center without any tendency to fall off. Certainly, in forecasting, no one expects to find the maximum velocities at 200 or 300 miles from the center. The two-part theory itself, although gradually reducing the velocity from a maximum at the boundary R to zero at the center, does not explain the existence of the central calm. (3) While the differential equation has two solutions which give some aspects plausible for this application, yet it is improbable that in such processes of

nature as the circulation of the air, there should be more than one law actually in operation. That the movement should suddenly jump from one system of forces to another is quite unlikely, unless cause can be shown for it. (4) In spite of skillful devices by which discontinuity in the rotation velocity was overcome, it is evident that there still remains a vertical discontinuity at the boundary, which becomes more and more pronounced with the increase in height above the surface, since $w = cz$. While it is hardly possible to actually observe the vertical components, yet the probabilities are that vertical motion sets in as soon as the isobars which surround the cyclone are closed up, and that all over this area there is a rising current. It may be laid down as a principle that where closed isobars exist there is an ascending or a descending current, according to the direction of the rotation. Where the isobars wander about without closing up, it may be assumed that there is no rising or descending air. In the case of cyclones this is confirmed by the general tendency of precipitation to occur over the entire region of the closed isobars. The preponderance on the eastern side over the western is due to the action of the general drift in the upper strata upon the circulation.

Hence, we conclude that while the Ferrel and the German vortices are each possible and may exist under certain conditions of boundary and distribution of heat, they do not agree with the cyclonic and anticyclonic circulation as given by the cloud observations of 1896-97. Although it is not possible to utilize the Ferrel vortex in further developments because the outer boundary is lacking, and though the German vortex, on the other hand, has apparently a closer application, yet even here it will be found difficult to avail ourselves of it without resorting to a modified method of analysis. We shall show, in part only, how this may be accomplished in the following papers, but the fact remains that the atmospheric circulation is usually too complex to be readily reduced to simple vortex motion of any kind. Hydrodynamic theories of stream lines must, on the other hand, be employed on a larger scale in the meteorology of the future than has been done in the past.

FERREL'S THEORY OF THE GENERAL CIRCULATION OVER A HEMISPHERE.

We can only briefly mention the principles which prevail in Ferrel's and in Oberbeck's solution for the circulation of the atmosphere over a hemisphere of the earth. In this case the boundaries are fixed, namely, the earth's surface, the plane of the equator, the topmost stratum of the atmosphere, and the pole of rotation. The heat distribution is such that the polar regions are cold and the Tropics warm. The primary idea adopted in the mathematical analysis is that of the so-called canal circulation, as, for example, when fluid in a long vessel with rectangular sides is artificially heated at one of its ends, so that the fluid rises at that end, falls at the other, moves in a horizontal direction from the warm end toward the cold end in the upper layers, but from the cold end to the warm end in the lower layers. In the same way the atmosphere is assumed to rise at the Tropics, move northward in the upper strata, fall in the polar zones, and flow southward along the surface of the earth. The effect of the contraction of the meridians, together with the rotation of the earth, is to introduce a complex torque effect, which causes the air to flow rapidly eastward in the temperate zones, especially in the upper strata, and westward in the tropical zones, especially in the lower strata. The general result is shown on fig. 14, for Ferrel's solution; and on fig. 15 the relative component velocities are given for Oberbeck's solution. These two methods of solution have some features in common, and also some of the results agree, and yet there is wide divergence in other respects, as will be indicated. The most conspicuous feature for us to note is that a neutral plane of velocity for the components u along the meridian is obtained, where there is no northward or southward velocity, but above it an increasing northward, and below it an increasing southward velocity proportional to the distance from this plane.

We shall have to compare this view with the results of the observations as given in the data of the year 1896-97. The main features of Ferrel's solution of the general cyclone are contained in the following extracts from pages 588-590, *International Cloud Report*:

Polar equations of motion applicable to the general circulation over a hemisphere:

$$200. \quad -\frac{1}{\rho} \frac{\partial P}{r \partial \theta} = \frac{du}{dt} - \cos \theta (2n + \nu) v + \frac{uv}{r}; \text{ where } \nu = \frac{v}{r \sin \theta} \\ -\frac{1}{\rho} \frac{\partial P}{r \sin \theta \partial \lambda} = \frac{dv}{dt} + \cos \theta (2n + \nu) u + \sin \theta (2n + \nu) w. \\ -\frac{1}{\rho} \frac{\partial P}{\partial r} = \frac{dw}{dt} - \sin \theta (2n + \nu) v - \frac{u^2}{r} + g.$$

Assumptions are made precisely analogous to those for the local cyclone, except that the temperature is expressed by the equation,

$$t = \Sigma A_s \cos s \theta, \text{ where,} \\ A_0 = 8.50^\circ. \quad A_1 = -1.75^\circ. \quad A_2 = -20.95^\circ. \quad A_3 = -1.00^\circ. \\ A_4 = -2.66^\circ.$$

The equations of general motion take the form,

$$397a. \quad -\frac{\partial \log P}{a \partial x} = \frac{du}{dt} - \cos \theta (2n + \nu) v + k u \\ - \frac{gh a}{r(1 + at)} 4 A_2 \sin \theta \cos \theta. \\ 0 = \frac{dv}{dt} + \cos \theta (2n + \nu) u + k v.$$

The second equation admits of integration between the pole and the plane of the equator for the entire rotating mass of air, with the resulting equation for the velocity v ,

$$v = \left(\frac{2}{3} \cdot \frac{1}{\sin \theta} - \sin \theta \right) r n.$$

If $v = 0$, $\theta = 54^\circ 44'$, and the latitude $\varphi = 35^\circ 16'$ where the velocity reverses direction at the surface.

The locus of $v = 0$, above the surface forms an arch over the equator, as shown in fig 14. This is analogous to the curve R of fig. 12 in the cold-center cyclone.

TABLE 13.—Theoretical west-east velocities.

Latitude ϕ .	v in miles per hour.
90°	+ ∞ eastward.
80°	+3807 "
70°	+1669 "
60°	+ 865 "
50°	+ 410 "
40°	+ 108 "
35° 16'	0 "
30°	— 100 westward.
20°	— 239 "
10°	— 320 "
Equator 0°	— 346 "

Professor Ferrel was governed in his method of integration by the theorem of the preservation of areas, $\omega v = \text{constant}$, depending chiefly upon the velocity v along the parallels of latitude, in order that the sum of the momenta might be equal to zero, $\Sigma mv = 0$, for the entire earth, which is a necessary result. However, it led to impossible velocities, v , as shown in Table 13, where excessive westward velocities prevail in the Tropics, and enormous eastward velocities in the polar regions. If we may assume that the location of the neutral plane is determined by the fact that half the air moves northward over it, and half the air southward under it, then the height of this plane should be about 6 kilometers = 3.7 miles above the ground, as given in Table 14.

TABLE 14.—Vertical diminution of pressure.

Height. <i>H</i> .	Pressure. <i>P</i> .	Per cent.	Height. <i>H</i> .	Pressure. <i>P</i> .	Per cent.
<i>km.</i>	<i>mm.</i>		<i>km.</i>	<i>mm.</i>	
0	760	100.0	8	280	36.9
1	671	88.3	9	247	32.6
2	591	77.8	10 (Cirrus)	218	28.7
3	523	68.9	11	193	25.3
4	461	60.7	12	170	22.4
5	407	53.6	13	150	19.8
6 (A. Cu.)	359	47.3	14	133	17.5
7	317	41.8	15	117	15.4

FIG. 14.—Ferrel's general cyclone.

Ferrel attempted to show that the excessive east-west velocities could be reduced to proper proportions by introducing a coefficient of friction, and considering that the sum of the moments Σmv_k above the neutral plane must be much greater than the Σmv_0k below that plane. The excess of energy $\Sigma mv_k - \Sigma mv_0k = E$, must be used up in overcoming the motion of the atmosphere, employing the term friction to include the forces that retard circulation by internal turbulent motion, or by the action of the adjacent discontinuous surfaces of the larger streams. It is evident, however, even supposing this theory

FIG. 15.—Oberbeck's component motions in the general cyclone.

correct, that this source of retardation is by no means sufficient to overcome the great amount of energy which must be consumed in motion to equalize the heat energy derived from the solar radiation in the Tropics. Professor Ferrel never executed

a complete integration involving all the component equations of motion, but merely discussed his several equations under different relative conditions, and thus drew out of them certain conceptions of the general circulation of the atmosphere which it was easy to show harmonized fairly well with many of the facts which were known to him at the time of his study, now nearly twenty years ago. It has become increasingly difficult, however, to believe that this solution is really as satisfactory as was then supposed.

OBERBECK'S SOLUTION OF THE GENERAL CIRCULATION.

Oberbeck attacked the same problem by a more complete analysis, and reached conclusions which in general accord with those of Ferrel, but differ from his in important particulars. He subdivided the total pressure of the atmosphere into partial pressures, and deduced a series of component velocities which could be computed by means of coefficients distributed at equidistant intervals throughout the atmosphere. An upper boundary was assumed for the atmosphere, but the solution was conducted in such a manner that this limiting stratum, whose height is H , could be changed in relation to the radius of the earth, R . The equations of motion were constructed for an origin at the center of the earth, while Ferrel's origin was on the surface, but the two systems of equations can be shown to be equivalent, so that the mathematical starting point is practically the same in both. Oberbeck held all the components together in one system, and hence, by not discussing them separately, could arrive at some conclusion which are really more instructive than Ferrel's. Yet it will be easily seen that these do not conform sufficiently well to the data of observation to be accepted as the complete solution of the problem.

Taking the component equations and notation given on pages 591–593 of the International Cloud Report, Oberbeck's solution for the component velocities is as follows:

South:

$$u = C 6 \cos \theta \sin \theta \frac{\sigma}{48} (6h^2 - 15h\sigma + 8\sigma^2).$$

$$u_3 = \frac{2n}{\kappa} R^3 \cos \theta \sin \theta (\nu_1 + 2\nu_2 - 7\nu_1 \cos^2 \theta) \frac{\sigma}{48} (6h^2 - 15h\sigma + 8\sigma^2).$$

East:

$$\nu_1 = D \sin \theta (1 - 3 \cos^2 \theta) \frac{\sigma}{480} (-9h^5 + 15h^2\sigma^3 - 15h\sigma^4 + 4\sigma^5).$$

$$\nu_2 = D 6 \cos^2 \theta \frac{\sigma}{960} (20h^2\sigma^2 - 25h\sigma^3 + 8\sigma^4).$$

Zenith:

$$w = C (1 - 3 \cos^2 \theta) \frac{\sigma}{8} (h - \sigma)(3h\sigma - 2\sigma^2).$$

$$w_3 = \frac{2n}{\kappa} R^3 [\nu_1 + 2\nu_2 - 6(4\nu_1 + \nu_2) \cos^2 \theta + 35\nu_1 \cos^4 \theta] \times \frac{\sigma^2}{48} (h - \sigma)(3h - 2\sigma).$$

$$h = \frac{H}{R} = \frac{1}{100}; \sigma = \left(\frac{0}{10}, \frac{1}{10}, \frac{2}{10} \dots \frac{10}{10} \right) h;$$

$$C = 0.5429 R^2; D = 0.00008532 R^4.$$

$r = R + Rh = R + H =$ radius of earth + height of atmosphere.

$$r = R + R\sigma = R + \frac{n}{10} Rh = \text{radius of earth} + \text{height of the}$$

stratum σ ; h is the maximum value of σ .

Assumed law of relative angular velocity, $\nu = \nu_1 \cos^2 \theta - \nu_2$.

Tables 15, 16, 17, give the relative coefficients of the component velocities in the direction of the meridians, the parallels of latitude, and in the vertical, respectively. By assigning values to H the different velocities may be computed.

I have not been successful in obtaining such velocities as will harmonize at all well with the known movements of the atmosphere, and I have, therefore, been led to distrust the

TABLE 15.

I. *Components on the meridians due to the rotation of the earth.*
 u = horizontal currents on meridians. $+$ = south, $-$ = north.

[illegible]

II. *Components on the meridians due to the relative motion of the atmosphere.*
 u_3 = horizontal currents on meridians. $+$ = south, $-$ = north.

$$\text{Coefficients of } \frac{2n}{\kappa} R^3 \frac{H^3}{R^3} = .1571 \times 10^{-9} H^3 = \begin{cases} .4062 \times 10^5 \text{ for } H=63700^m. \\ .4062 \times 10^2 \text{ for } H=6370. \end{cases}$$
[illegible]

TABLE 16.

1. *First components on the parallels due to the rotation of the earth.*
 v_1 = horizontal currents on parallels. $+$ = east, $-$ = west.

[illegible]

TABLE 16—Continued.

11. *Second components on the parallels due to the rotation of the earth.* v_2 = horizontal currents on parallels. + = east, — = west.

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Coefficients of $D \frac{H^5}{R^5} = .8532 \times 10^{-4} \frac{H^5}{R} = \begin{cases} 1.405 \times 10^{13} \text{ for } H=63700^m. \\ 1.405 \times 10^8 \text{ for } H=6370. \end{cases}$										
$\sigma=1.0$	.00000	+.00315	+.00565	+.00702	+.00706	+.00592	+.00404	+.00206	+.00056	.00000
.9	.00000	+ 305	+ 547	+ 680	+ 683	+ 573	+ 391	+ 199	+ 54	.00000
.8	.00000	+ 276	+ 495	+ 614	+ 618	+ 518	+ 354	+ 180	+ 49	.00000
.7	.00000	+ 231	+ 415	+ 515	+ 518	+ 435	+ 297	+ 157	+ 41	.00000
.6	.00000	+ 179	+ 321	+ 398	+ 401	+ 336	+ 229	+ 117	+ 32	.00000
.5	.00000	+ 125	+ 225	+ 279	+ 281	+ 235	+ 161	+ 82	+ 22	.00000
.4	.00000	+ 76	+ 136	+ 169	+ 170	+ 142	+ 97	+ 50	+ 13	.00000
.3	.00000	+ 37	+ 67	+ 83	+ 84	+ 70	+ 48	+ 24	+ 7	.00000
.2	.00000	+ 13	+ 24	+ 29	+ 29	+ 25	+ 17	+ 9	+ 2	.00000
.1	.00000	+ 2	+ 4	+ 4	+ 5	+ 4	+ 3	+ 1	0	.00000
.0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

TABLE 17.

I. *Vertical components due to the rotation of the earth.* w = vertical currents. + = ascending, — = descending.

Coefficients of $C \frac{H^4}{R^4} = .5429 \frac{H^4}{R^2} = \begin{cases} 2.203 \times 10^5 \text{ for } H=63700^m. \\ 2.203 \times 10 \text{ for } H=6370. \end{cases}$										
$\sigma=1.0$	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	— .0244	— .0233	— .0202	— .0152	— .0093	— .0029	+.0030	+.0079	+.0111	+.0122
.8	— 448	— 428	— 370	— 280	— 170	— 54	+ 56	+ 145	+ 204	+ 224
.7	— 588	— 562	— 486	— 368	— 224	— 70	+ 74	+ 191	+ 268	+ 294
.6	— 648	— 619	— 535	— 405	— 247	— 78	+ 81	+ 210	+ 295	+ 324
.5	— 624	— 596	— 515	— 390	— 237	— 75	+ 78	+ 202	+ 284	+ 312
.4	— 528	— 504	— 436	— 330	— 201	— 63	+ 66	+ 171	+ 240	+ 264
.3	— 378	— 361	— 312	— 236	— 144	— 45	+ 47	+ 123	+ 172	+ 189
.2	— 208	— 199	— 172	— 130	— 79	— 24	+ 26	+ 68	+ 95	+ 104
.1	— 64	— 61	— 53	— 40	— 24	— 8	+ 8	+ 21	+ 29	+ 32
.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

II. *Vertical components due to the relative motion of the atmosphere.* w_3 = vertical currents. + = ascending, — = descending.

Coefficients of $\frac{2n}{\kappa} R^3 \frac{H^4}{R^4} = .1571 \times 10^{-9} \frac{H^4}{R} = \begin{cases} .4062 \times 10^3 \text{ for } H=63700^m. \\ .4062 \times 10^{-1} \text{ for } H=6370 \end{cases}$										
$\sigma=1.0$	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	+.0502	+.0444	+.0294	+.0106	— .0050	— .0128	— .0119	— .0053	+.0020	+.0050
.8	+ 927	+ 820	+ 542	+ 197	— 92	— 237	— 220	— 97	+ 37	+ 93
.7	+ 1217	+ 1077	+ 712	+ 258	— 121	— 311	— 289	— 127	+ 49	+ 122
.6	+ 1341	+ 1187	+ 785	+ 285	— 133	— 343	— 319	— 140	+ 53	+ 134
.5	+ 1294	+ 1145	+ 757	+ 275	— 129	— 331	— 307	— 135	+ 52	+ 130
.4	+ 1093	+ 967	+ 639	+ 232	— 109	— 279	— 260	— 114	+ 44	+ 110
.3	+ 782	+ 692	+ 458	+ 166	— 78	— 200	— 186	— 82	+ 31	+ 78
.2	+ 430	+ 380	+ 251	+ 91	— 43	— 110	— 102	— 45	+ 17	+ 43
.1	+ 132	+ 117	+ 77	+ 28	— 13	— 34	— 31	— 14	+ 5	+ 13
.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

validity of this "canal theory" of the circulation, as explained in my report. An inspection of the coefficients, Table 15, shows that in the case of the meridian components, u , the upper northward and the lower southward circulation as deduced for the rotating earth, is somewhat modified by the component u_3 depending on the motion of the atmosphere relative to the earth itself. The component v along the parallels of latitude, Table 16, differs radically from Ferrel's result, and it should be carefully noted. Oberbeck divides the eastward drift in higher latitudes from the westward drift in lower latitudes at the

parallel 35° . The eastward v is a maximum in the neighborhood of latitude 60° , $\theta = 30^\circ$, but vanishes at the poles. This is exactly contrary to Ferrel's result, which made the velocity v a maximum at the pole, before the assumed modification by friction was applied. Oberbeck makes the westward drift a maximum at the plane of the equator, which is certainly not in conformity with the observations. He also makes the westward velocity increase at the equator from the surface to the upper boundary, and show no sign of a reversal from westward to eastward at a moderate elevation, as is generally believed to be the fact, judging from certain well known motions of the air observed in the Tropics.

The United States Weather Bureau has been conducting a series of nephoscope observations in the West Indies for the past three years, and it is hoped that the discussion of these observations, soon to be undertaken, will give us some definite information on this important point.

The second term v_2 modifies v_1 , but the two combined, $v = v_1 + v_2$, sustain the conclusions just mentioned. This feature of Oberbeck's solution is so far from conforming to the observed motions of the atmosphere that it seems to me to be inferior in value to Ferrel's for the Tropics. Ferrel's arch over the Tropics, shown in fig. 14, is probably a fact, and if this is so, then the only serious modification required in Ferrel's treatment is to show how the excessive eastward drift in the mid-latitude and polar zones can be effectively checked. It is evident that there must be a large amount of energy available for use in the construction of local cyclones and anticyclones, and that there is, therefore, no pressing need to refer the

energy of these motions to any local supply of heat, as is done by those who extend to cyclones the theory of the latent heat of condensation from precipitation originally devised by Espy to explain cumulus clouds and thunderstorms. The components w and w_3 , Table 17, show that there is an ascending current in the Tropics, and a descending current in the higher latitudes. Thus, as the result of the theoretical discussion in general, the canal theory has several of its features verified, and yet there are serious discrepancies inherent in both Ferrel's and Oberbeck's solutions.

My statement has suggested by implication that there exists an important principle which has been neglected by these meteorologists. They have each discussed the general and the local cyclones as if they were in a sense *independent of one another*, since separate sources of heat energy are assigned to each, and two characteristic laws of circulation are deduced therefrom. It is much more natural to suppose that these two systems are mutually interdependent, and that the excess of energy of the general cyclone is transformed into the driving forces of the local circulation; also, that the acquired motion of the local cyclone reacts upon and retards the excess of motion of the general cyclone in the temperate zones. The subject becomes, however, excessively complex, and I can only attempt to sketch in general terms in my next paper an outline of this view, hoping some other time to be able to supplement it with a more suitable mathematical analysis, when the study of the observations now in hand has been advanced more nearly to completion.

V.—RELATIONS BETWEEN THE GENERAL CIRCULATION AND THE CYCLONES AND ANTICYCLONES.¹

UNEQUAL DISTRIBUTION OF CYCLONES IN NORTH AMERICA AND EUROPE-ASIA.

We have arrived at the following proposition as the result of the discussion of Ferrel's and Oberbeck's analysis of the general and local circulation, that the general cyclone and the local cyclones and anticyclones have been treated almost independently of one another, while in fact the imperfect results of the theory and the modern observations both indicate that these two classes of movement should be analyzed in close relation with each other. The evidence compels us to regard both these circulations as the common effect of the readjustment of the thermal equilibrium, which is disturbed by the radiation of the sun falling on the tropic zones, and the true meteorological problem is to trace out the successive stages in the process of this interaction through the resulting currents which circulate in the atmosphere. The results contained in this paper apply especially to the North American Continent, and it is hardly to be expected that the details will be found the same in all the other regions of the earth. Indeed there are several reasons for believing that this continent is the peculiar theater for the interchange between the heat and the cold of the Northern Hemisphere, and that the Euro-Asian Continent plays a very different role in the meteorological economy of this hemisphere. For it is well known (1) that while the American Continent is the place for the profuse generation of cyclones, Europe is the region for their dissipation, and in Asia very few cyclones occur except along the ocean borders; (2) that the velocity of motion of the atmosphere generally is about twice as great over North America as over Europe. This points to a profound difference between the actions of the atmosphere in these two regions, but one cause of it at least is easily perceived. It has been shown that the currents which are especially concerned in forming cyclones are contained for the most part within 2 or 3 miles of the ground, though their accompanying effects may extend much higher. Hence, any barriers of elevated ground, as mountain ranges, which tend to deflect the flow of the lower strata, must strongly influence the formation of the cyclones themselves, if they are to be referred to the counter flow of long horizontal currents of different temperature rather than to local vertical convective currents.

The great range of the Himalaya Mountains stretching east and west is such a barrier to the flow of the tropical and polar currents in that region, and the result is that true cyclonic movements are almost excluded from the interior of Asia. On the other hand the Rocky Mountain range, stretching north and south along the western districts of North America, favors the counter flow from the Tropics and the polar regions by deflecting the westward current of the Tropics toward the north, and the eastward drift of the higher latitudes toward the south. The same tendency is favored by the location of the high pressure area to form over the middle Atlantic Ocean, while the Rocky Mountain range breaks through the midst of it. The result is to produce a powerful anticyclonic center of action over the Atlantic Ocean, which maintains a strong northward component from the West Indies toward the interior of the continent. At the same time the American Continent causes the isobars and isotherms to loop southward, and thus in consequence to draw the Siberian atmosphere in a direction nearly parallel to the Rocky Mountain range. These physical conditions are a constant incentive to the formation of countercur-

rents which meet on the Canadian and United States territory, with the result that 75 per cent or 80 per cent of the storms of the Northern Hemisphere are generated in these districts. It is not necessary for maintaining the temperature equilibrium of the hemisphere that the interchange of heat and cold should occur so as to have a uniform distribution over all portions of it, because if there is an excessive interchange in any place, as in North America, the general movements of the atmosphere will soon transfer the effects to all other parts. Keeping these facts in mind will facilitate an understanding of the views which will be briefly described as follows:

CRITICISM OF THE CANAL THEORY OF THE GENERAL CIRCULATION.

The immediate problem before us is this: To what extent is the canal theory of the general circulation over a hemisphere correct, and in what direction must it be modified to conform to the modern observations? Ferrel derived the following equations from a discussion of the first equation of 397a for the approximate velocities and gradients in the strata of the upper atmosphere:

$$408a. \text{ Velocity; } v = v_0 - \frac{0.016}{(2n + v)} \cdot \frac{gh}{r} \cdot \frac{A_2 \sin \theta}{(1 + at)};$$

$$409a. \text{ Gradient; } G = G_0 - \frac{0.00001327}{\cos^2 i} \cdot \frac{A_2 \sin 2\theta}{(1 + at)^2} \cdot \frac{P}{P_0} h;$$

when v_0, G_0, P_0 are the values at the surface, and v, G, P , the values at the height h . Since A_2 is negative, -20.95° , it follows that v is greater than v_0 , and G is greater than G_0 , so that the eastward velocity and the meridional gradient velocity increase with the height. The relative eastward velocity is shown on fig. 16. The interchanging velocity along the meridians is northward above the neutral plane and southward below it as given on fig. 16.

FIG. 16.—Ferrel's component currents by the canal theory.

The canal theory requires that the flow of the atmosphere should be in unbroken stream lines conforming to these precepts. The result of the principle of preservation of areas, $v \sin \theta = \text{constant}$, as applied to the axis of rotation of the earth, is that the velocity v is excessive near the poles, and the gradients of the upper strata much too large. Ferrel sought to escape this difficulty by evaluating $\sum m (v - v_0) k$ for the upper strata with a northward $-u$, and $\sum m (v - v_0) k$ for the lower

¹ Reprinted from the Monthly Weather Review for May, 1902.

strata with a southward $+u$, and assuming that the difference of momenta is equal to the retardation of the eastward drift by the frictional resistance. It is known, however, that the frictional coefficient is a very small factor, and not capable of producing the required retardation.

Furthermore, the modern observations show, as in Paper III,² that the northward $-u$, and the southward $+u$, components are not distributed as the canal theory requires, but that there are approximately equal currents flowing northward and southward on the same levels, which reduces the difference of momenta to zero, and is fatal to the frictional theory of retardation. The counter flow of horizontal currents on the same levels, most powerful in the strato-cumulus level, does, however, constitute a dynamic mechanism quite capable of retarding the eastward drift, so as to produce the observed moderate eastward velocities of the temperate zones. This is the true source of the energy consumed in the motions of cyclones and anticyclones. Compare fig. 17.

FIG. 17.—Bigelow's component currents from the Weather Bureau observations.

The general theory which has already been mentioned in my previous paper—*Storms, Storm Tracks, and Weather Forecasting*, Bulletin No. 20, Weather Bureau, 1897—is practically as follows: There is a sheet of the atmosphere flowing quite steadily eastward over the United States at the levels from 2 miles upward. Beneath this flows north and south, quite independently of the upper drift, a series of countercurrents which are cold and warm, respectively. The interaction of these currents sets up the anticyclonic and cyclonic whirls chiefly by dynamic action, the former causing a downward component and the latter an upward component. The cyclone receives its supply from the northwest and from the south, and thus discharges great masses of air through itself, so that the cyclonic configuration constitutes a type of persistent circulation in a vortical form of stream lines. This gyratory rotation lifts the air from one level to another, purely by its mechanical action, and thus raises the mass from the strata of lower to those of higher eastward velocity. This rapid change of eastward motion, if imparted to the air raised up, can take place only by an interchange of inertia, that which is gained by the rising air being lost by the upper eastward drift. Also the gyration of the lower strata causes deflection in the eastward drift by the composition of forces, as must be the case, since the method followed in our discussion has been to subtract the normal west-east and north-south velocities from those actually observed, the residuals being the circular cyclonic movements that are now described.

The point of view for the consideration of the theory of the general cyclone is thus considerably altered. (1) Instead of

a circulation as in a canal, northward above and southward below, we find that the interchanging motion is largely confined to the lower strata, by means of currents not flowing above one another, but on the same level. (2) Instead of the momentum $m(v_0 - v)$ being determined by the difference between the eastward flow at different levels of the same latitude, we prefer the statement that cyclonic gyrations are produced by the counter flow of independent streams, and that the rapid eastward drift is retarded by mechanical inflows, from above toward the base of the anticyclone, and from below through the stream lines of the cyclone into the eastward drift. The energy upon which cyclones and anticyclones depend for their activity is to be traced to a different source from that generally assigned to it by meteorologists. The common theory is that the cyclone is due to some form of vertical convection, caused by overheating a local region, and by the latent heat produced in precipitation. Our theory would more naturally depend upon horizontal convection, by means of which temperature gradients thousands of miles in extent produce comparatively shallow streams, which flow from the north and from the south, and sustain them for considerable intervals of time. The upward and the downward discharges, together with the rotary components which make the sinuous flow of the air in the upper levels, practically tie together the upper and the lower strata, retard the eastward drift in the higher strata, and accelerate the eastward motion in the lower. This effect is readily perceived in the eastward propagation of high and low pressure areas over the United States, which is the basis of our system of forecasting and renders it possible.

It is by no means to be concluded that by suggesting this modification of the Ferrel theory, any intention exists of not recognizing fully the fact that it remains substantially correct in some of its features. There exists the eastward drift throughout the middle latitudes and the westward drift in lower levels of the tropical zone. But there is yet another reservation to be introduced. The heating of the tropical belt raises the isobars adjacent to the equator so that they slope toward the poles, and to such an extent that they almost exactly counterbalance the deflective force $2uv \cos \theta$, which is directed southward. It may properly be assumed from this that in the upper strata the directions of the isobars, the isotherms, and the stream lines of the wind motion are very nearly parallel to each other, if not coincident. The friction is evidently small, judging from theoretical conditions, and from the results of the observations, or else this could not be the case.

There is another important deduction to be drawn from this discussion regarding the flow of currents from the Tropics toward the poles in the lower strata. According to the Ferrel theory, the overheating of the Tropics is relieved by the upward expansion and overflow, but in accordance with the present view the tropical congestion is relieved by irregular streams which flow outward from the lower levels. This being the case, the poleward gradients in the upper levels are called upon to sustain much less pressure from the deflection, and evidently the tendency to excessive eastward velocities is much diminished, just because the equatorial lift of the strata can not be so great, since the expansion upward, as stated, leaks off sideways in the lower levels. The eastward drift does not therefore increase to excessive values for these two reasons: (1) The tropical strata are not elevated up to the theoretical amount because of the escape of the currents poleward not very far from the surface of the ground; (2) the eastward drift is diminished by the operation of the vertical discharges between the lower and the higher levels produced by the purely mechanical vortex motion in the cyclones and the anticyclones.

The evidence before us is to the effect that the heating of the atmosphere is generally confined to a layer less than 5,000 meters, or 3 miles, thick. It is not intended to allude now to the annual range in temperature with the sun's change in

² Monthly Weather Review, March 1902, Vol. XXX, p. 117.

latitude, but rather to the shorter periods of only a few days length which contribute to the impulse of streams from the south. There are several reasons which lead to this conclusion: (1) The trend of the preceding argument has been to show that the readjustments of disturbed temperature equilibrium take place in the lower layers of the atmosphere by means of rather spasmodic impulses, controlled partly by the temperature energy in the tropic and the polar regions, partly by the distribution of land and ocean temperatures, and by the relative radiation which takes place from them in the winter and summer season, respectively. (2) Half of the mass of the atmosphere is contained below the 5,000-meter level, and this is the layer within which the greater part of the aqueous vapor is also collected, since the vapor contents of the higher levels is in the form of fine ice crystals, which drift eastward, encircling the earth, and perhaps seldom finding their way to the ground. The fact that the dust of the Krakatoa volcano was thus carried about the earth for two or three years shows that the upper current has a history of its own, to a considerable extent independent of the 3-mile layer nearest the earth. Now, the important part which the aqueous vapor plays in the absorption of heat is well understood, and it depends upon the very high latent heat of water, which is 606.5 calories per kilogram at 0° C. The evaporation and condensation of water in precipitation is certainly confined to the lower stratum, and hence it is in harmony with this view to limit the effective heating of the air by the sun to the lowest 3 miles. (3) The diurnal variation of the temperature at the surface of the ground takes on a wide range. The temperature is above the normal in low areas, in the summer season, and in the middle of the day; it is below the normal in high areas, especially in the case of cold waves in the winter, and in the night-time. The range at the ground generally amounts to 10° or 20° F., but diminishes upward with the height and disappears at the 5,000-foot level, or even considerably below that height. This is shown very clearly in the study of the changes in the vertical temperatures, as explored by means of balloon ascensions, where the 5,000-foot level marks the convergence of the lines which represent the gradients in the forenoon and afternoon. The range in the United States is usually greater than in Europe at the ground, owing to the more pronounced nature of the cold and warm waves that move eastward over that region, but the evidence is that the diurnal lines converge at 2 or 3 miles above the ground. (4) The fact that the great eastward drift of the upper levels is underrun by a series of comparatively thin currents, which move about in every possible direction, shows that the disturbance of equilibrium is local and confined to a shallow skin near the ground, the most rapid currents belonging to the cumulus and strato-cumulus levels, which also implies that the upper regions of the atmosphere are much less affected than the lower. (5) The same conclusion is indicated very clearly by the seasonal change in the drift of the high areas over the northwestern portions of the United States, from the northwest in winter and from the southwest in summer, in conformity with the location of the permanent high areas in winter over the continent and in summer over the ocean.

From these considerations it seems evident that the upper atmosphere is but slightly disturbed in its temperature equilibrium by the effects of the solar radiation, but the solar rays pass through it with comparatively little absorption, while the larger percentage of the heat retained in the lower strata, is due to a change of the wave length. This conclusion is very important for two reasons: (1) It shows why the general circulation of the atmosphere prescribed by the Ferrel-Oberbeck theory does not seem to be confirmed by the observations. (2) It also indicates where the energy comes from which is finally expended in the generation of cyclonic circulations and in the retardation of the eastward drift by the agency of inertia rather than by friction; for if the total

amount of energy falling upon the Tropics does not expend itself in an upper poleward current, because the higher strata retain a temperature of equilibrium almost undisturbed, then this energy must give rise to a series of comparatively small currents moving poleward in the lower strata—a fact which is abundantly confirmed by the observations. Also, if the friction of the upper atmosphere upon itself is very slight, then there will be but little retardation to an excessive eastward drift, tending under a steady force to have a constant acceleration. The only other available agency which will produce the same effect is the intrusion into the higher strata from the lower, or vice versa, of air moving eastward with a different velocity, which must suddenly be subject to acceleration. The discharge of the product passing through cyclonic circulations is perfectly fitted to perform this office. Hence, the theory here expounded consists of two parts as regards the eastward drift: (1) The upper poleward gradients are not built up to the amounts supposed by Ferrel, because of the lateral escape poleward of currents from the tropical belt; (2) The agency of friction as a retarder is replaced by the interchange of inertia derived from a compound circulation, the sources of the separate parts having different and independent causes. It has been important to thus carefully clear the ground for the theory of local cyclones, which will be advanced to take the place of the type proposed by Professor Ferrel on the one hand, or of that advocated by the German school on the other hand. These two theories are not in harmony with each other, and neither of them seems to be in agreement with the observations. It is very evident that the six assumptions which have been made in order to pass from the general equations of motion, 200, to the working system employed by Ferrel, 397a, must be carefully revised before we can expect to put this branch of meteorology upon a correct working basis. In particular it is not suitable to omit the variations of temperature in longitude, because in so doing the turbulence of the lower strata and the alternate streams which are implied in this variation profoundly modify the general and the local circulation. It is also necessary to correct the statement regarding the friction as the special agency of retarding accelerated flows and substitute, or rather add thereto, the inertia of currents which are rapidly changing their velocities. The conflict of turbulent countercurrents, especially at a short distance above the ground, must be rigorously considered in studying the resultant effects. Likewise the direct application of the law of conservation of areas passed over by the rotating radius vector can not apply immediately to the lower strata, though it may be much more nearly correct above the 3-mile level.

MODIFICATION OF THE CANAL THEORY.

In consequence of these considerations it seems necessary to modify the canal theory to such an extent as to be practically equivalent to an abandonment of it. If this is done it is important to trace out a chain of circumstances which will give a more correct account of the general and the local circulation of the atmosphere. The canal theory is very artificial, depending as it does upon a simple laboratory experiment, in connection with an obvious analysis of the general equations of motion. If this theory is to be preserved, then we must be assured that the atmosphere does in fact traverse the circuits prescribed, for it has been commonly assumed to conform with the facts of observation. As has been shown, observation does not bear out the requirements of the theory, and I shall, therefore, attempt to trace out in a descriptive way the circulation as it is developed over the North American Continent. The reduction of this kinematic picture to the corresponding mathematical form of dynamics is a task of very great difficulty, as may readily be inferred.

We may conceive the tropical strata to be elevated by thermal expansion relative to the polar regions, so that there is a certain average gradient slope and corresponding west-east

velocity which is in equilibrium with it in accordance with the usual equations. If the canal theory of circulation is in operation we have poleward gradients in the upper strata, and equatorward gradients in the lower strata. If, instead of maintaining this circulation, there is an escape of currents from the Tropics in the lower strata in a poleward direction, then the gradients of the canal theory diminish and become much more moderate in consequence of this release of tension. Every such leakage current from the Tropics causes a break or fault in the gradient, and this must be attended by a corresponding deflection of the eastward current, through the operation of the deflecting force at right angles to it, due to the earth's rotation. Referring to the scheme of self-regulation of the circulation by the rise and fall of the gradients, fig. 18,

FIG. 18.—Scheme of self-regulation of the circulation by the rise and fall of the gradient.

we may consider the average gradient in the strato-cumulus level. When in its mean position, as determined by the whole set of natural circumstances which produce it, the isobars and the wind with a given direction and velocity are all practically in coincidence. The velocity is just enough to maintain the slope of pressure as measured by the gradient. If the thermal expansion of the Tropics increases, the gradient slope is elevated, the eastward velocity is increased, and this acceleration will continue to advance till an excess of energy secures for itself a way of escape. This increase of velocity is probably checked as follows:

From the Tropics in the lower levels, that is in the cumulus and strato-cumulus strata, a stream of air breaks away from the canal circuit, and pushes northward in some irregular course. This evidently causes a break in the gradient surfaces, such as is indicated, for example, by the dotted lines of fig. 18, and the slope of pressure which held the eastward drift in its position gives way. The action of the deflecting force due to the earth's rotation bends the current southward as is indicated, and there is thus made the beginning of an anticyclonic local movement. In the lowest levels, from cumulus down to the surface, the break in the average gradients may be so pronounced as to offer but little check to a complete anticyclonic gyration, such as appears at the surface of the earth. This deflection of direction causes a whirl and absorption of energy in the strata affected through the local interchange of inertia, and slows up the eastward drift by the vortex action which is produced with a downward component. The alternate rise and fall of the gradients, and the attendant anticyclonic gyrations, mark the successive efforts at self-regulation which the atmosphere as a thermal engine imposes upon itself. These continental pulsations are shown on the weather maps as the procession of high and low pressure areas which traverse the middle latitudes. In consequence of the superior velocity of eastward motion in the strato-cumulus level, this region is the first to feel the decline in gradient strength, due to the tendency of a stream to escape from the Tropics, as from the Gulf of Mexico over the United States. It thus happens that

the anticyclone is not only larger in area, but it also generally precedes the cyclone in its formation. There are numerous instances in which the anticyclone overspreads the United States, while there is no important cyclone in connection with it, except possibly some depression or irregular action along the edges of the high area. Usually, however, incipient cyclones increase in intensity from these small beginnings, and they may even seem to drink up and exhaust the air which is flowing in anticyclonic circulation. This is the reason why I have heretofore described the anticyclone as preceding the cyclone in efficiency, and have thus reversed the order of action as taught by Professor Ferrel in his well-known theory.

The warm stream from the south is deterred from mingling with the cold anticyclonic air by the difference of its temperature, and the result is that the eastern side of the anticyclone, which flows southward, and the warm escape current from the south, flowing northward, compose two counter currents. These two currents together generate the cyclone, which is a vortex with an ascending central velocity. The gyration is produced by the action of the two independent streams acting like a couple, since they each depend upon separate gradient systems for their own mechanical pressures. It is not a gyration due to frictional impact, but rather to the steady pressure on the arms of a couple, since the streams are driven by independent gradients, and are held apart by having different temperatures in the two separate currents. The air is thus raised from the lower to the higher strata in great masses, by circulating through the configuration of the cyclone, and this, too, produces a retardation of the eastward drift by an immediate interchange in the inertia, since there is a quick mingling of air having different directions and velocities. Examples of this action can be seen by studying the Charts 20 to 35, inclusive, of the International Cloud Report. Furthermore, the motions of the atmosphere result in placing strata of air having different temperatures together, side by side, so that the surface isotherms are directed from the southwest to the northeast, and in consequence the northwest portions of a cyclone are cold and the southeast portions are warm. This does not conform to the requirements of either the Ferrel or the German theories, which demand a warm local center for the generation of cyclones. Yet if two such masses of air lay alongside while they are of different temperatures, an interchange of heat contents will take place locally between them, and thus the streams will interflow in such a way as to strengthen the cyclonic gyration. This will be accompanied by distinct stratification of the air currents in the local cyclones, such as is observed in the kite and balloon ascensions, since the air of different temperatures is drawn out into thin ribbons having large discontinuous surfaces, which are favorable to the interchange of heat. It is frequently found that a great anticyclonic area as it approaches the ocean, although attended by no cyclone, will yet suddenly cause a violent whirl to form on its edge by the mere action of these adjacent masses of different temperatures. Such local storms are sometimes formed on the Atlantic coast during a single night, and they may cause vortices with hurricane velocities on the coast. The line of junction of these warm and cold currents, along the southern and southeastern parts of the low area, is the locus of the formation of the majority of the tornadoes of the United States, the counter flow setting up the gyration, which is converted into a genuine columnar vortex, through which the heated air of the lower strata escapes into the colder strata of the higher levels. The hurricanes of the West Indies similarly form along the places of the counter flow between the Atlantic high area and the southeast trade winds when at their extreme northern limit, as in August to October. The vortex then travels westward and skirts around the periphery of the high area until it is absorbed finally in the eastward drift of the higher latitudes. Such dynamic intermingling of the general and the local circulation is, therefore, not only

in accordance with observations, but it is a suitable substitute for the defective canal theory of the general circulation, and also for the untenable theory of the local cyclones and anticyclones, supposed to be dependent upon the central heat produced by condensation of the aqueous vapor of precipitation. This view is attended, on the other hand, by the following disadvantage: That while the canal theory and the warm center cyclone theory lend themselves readily to mathematical treatment and to analytic solutions of considerable elegance, we are obliged to substitute for them an irregular system of stream lines in the lower strata, not at all readily put into mathematical forms. This turbulent circulation, with its self-adjusting government of the eastward flow, its interaction between the general and the local vortices, its numerous subordinate phenomena, such as tornadoes, hurricanes, and cyclones, is easy to comprehend, but hard to analyze mathematically into the exact dynamic forces of equilibrium. It is possible to construct several special typical configurations for each district of the earth, as was done for the United States in Charts 20-35, inclusive, of the International Cloud Report, and then draw the stream lines, with their velocities, in order to prepare for the computation of the dynamic forces involved. This is the true meteorological problem of the future.

THE STRUCTURE OF THE ANTICYCLONE.

We will now examine a little more closely the structure of the anticyclone and the cyclone as given by observations for the sake of the analytical problems presented by their configuration. It has been claimed by meteorologists that there is a southward component in the middle strata of the north temperate zone toward the high pressure belt, but a northward component in the upper strata and another northward component near the surface, as is indicated on fig. 13, Paper IV³. The observations of 1896-97 do not, however, give such a distribution of the mean components, for they show that there is a very small average drift northward in all strata, increasing slightly with the height above the surface. That is to say, the atmosphere in the eastern and central United States drifts northward a very little and thus supplies part of the air that descends into the anticyclones through the upper strata. We have indicated how the leakage in the lower strata from the Tropics in part replaces the air which descends in the anticyclonic areas, and it is assumed that the small residual northward drift complements the amount that is required to fill the anticyclonic areas. The downward vortex, therefore, draws in a portion of the air passing through it from the upper strata, as a consequence of the gyration induced through the countercurrent action, and therefore the feeble northward component of the circulation of the higher strata seeks the surface practically in the middle latitudes, before arriving at the polar zone, through the mechanism of the local vortices. Hence, it follows that there is little cause for the formation of a general anticyclone close to the pole itself, which Ferrel assumed to exist; the result of Kimball's discussion in the MONTHLY WEATHER REVIEW, September, 1901, goes to show that this movement only feebly exists, and is in conformity with this exposition of the general circulation. Therefore, the air that descends in a local anticyclone comes from two sources, the leakage currents from the Tropics in the lower and middle strata and the feeble northward drift in all strata, especially the higher.

There is one feature of the anticyclonic vortex which may be mentioned, though it belongs more properly to an analytic treatment of that circulation. The anticyclonic components of fig. 6, Paper III⁴, show that we are not dealing with a pure form of vortex. The two possible laws are typically the para-

bolic $\frac{v}{\omega} = \text{constant}$, and the hyperbolic $v\omega = \text{constant}$.

FIG. 19.—Mixed system of hyperbolic and parabolic components.

According to the parabolic law $\frac{v}{\omega} = \text{constant}$, the circulation causes simply a depression in the center of the gyrating fluid; according to the hyperbolic law $v\omega = \text{constant}$, there is a vertical component of circulation as in ordinary vortices. In the observed anticyclonic components the velocities v are about equal to each other on the I, II, III, circles, and it seems to me that this can only happen if there is a mixture of these two laws of motion. Thus we may divide the observed components v_I, v_{II}, v_{III} , into two parts by a diagonal as shown on fig. 19. The upper components represent the parabolic, and the lower the hyperbolic portions. This is physically necessary for the following reason. The anticyclone is formed by large currents of air moving in more or less independent streams on its outer portions, while only curling offshoots reach its central parts; this would produce the pure parabolic components only. But, through imperfect pressure gradients there is also near the center a true downward component of circulation, and this can be supplied only by the action of hyperbolic components, that is to say, of a simple vortex motion. Hence, the general anticlockwise movement of the anticyclone, strongest on the outer circles, has accompanying it a true downward or vortex component which lengthens the components u in the central portions. If this is correct, one sees an additional reason for holding that Ferrel's explanation of the anticyclone is impracticable, and also that the reversing of the cyclone to make an anticyclone, as proposed by Oberbeck and Pockels is not warranted. We need very accurate observations to settle so difficult a point of pure theory, but I can not at present see any other satisfactory explanation of the gyratory components derived from the Weather Bureau observations.

STRUCTURE OF THE CYCLONE.

In Table 18 we give the results of cloud observations in the United States.

In the following example the relations which should exist in a pure vortex are deduced for comparison with the data given under low areas in Table 18.

Taking the inward radial velocity $u = -1.25$, at the distance 1,250 kilometers, assuming $\lambda = .000100$ for $\theta = 46^\circ 17'$, and $c = .000002$, also $k = .000050$, their introduction into the several formulæ gives the values found in Table 19.

An account will be given of the derivation of the formulæ in Paper VI.

It is seen that the rotational velocity v is about the same as that given by the observations up to the circle whose radius is 150 km = 93 miles from the center, as seen under v_2 of Table 18. The values of the radial velocity agree fairly well, if we admit that the observations may be somewhat imperfect for this component up to the region of the inner circle I, whose radius is 250 km. There the component u is much larger than expected in the upper strata, and this indicates some opposition to the free development of the vortex near the core. It is presumed that this implies a struggle to intrude into the rapid eastward drift, accompanied by a broadening of the vortex tube through the resistance, v being smaller than it should be and u greater, making the angle of the inclination i much greater than it ought to be in pure vortex motion. As the computation of the vortex goes to the height of 10 miles, far beyond the altitude to which the ordinary cyclonic motion penetrates, this being only 3 or 4 miles in the moderate movements of the air, it is seen that v should theoretically attain

³ Monthly Weather Review, April, 1902, Vol. XXX, p. 166.

⁴ Monthly Weather Review, March, 1902, Vol. XXX, p. 117.

enormous velocities very near the center. The difference between these and such as are actually observed may be regarded as measuring the energy expended in breaking up the cyclone in the higher levels, which can be balanced only by retarding the movements of the general cyclone. The vertical velocity w is extraordinarily small from the bottom to the top, and in a measure justifies the method of discussing the motions of the air in the cyclone as a case of horizontal movement. It is impossible theoretically, however, that such a cyclonic motion without a vertical component should exist at all. The fact is that a slow vertical movement is acting over the very large area covered by the cyclone, and is sufficient to carry off all the air which flows into it through a thin disk at the outside in horizontal directions toward the center. The checks ϖv and $\varpi w = -2uz$ hold throughout the cyclone, thus proving that our data are at least approximately correct. The angle i , between the tangential direction and the current, theoretically becomes very small within 300 miles of the center. It

TABLE 18.—Anticyclonic and cyclonic velocities at each 1,000-meter level. (Copy of Table 126 International Cloud Report.)

Cloud forms.	Height in meters.	High areas.						Low areas.						Velocities in the general cyclone.		
		I		II		III		I		II		III				
		u_2	v_2	u_2	v_2	u_2	v_2	u_2	v_2	u_2	v_2	u_2	v_2	u_1	v_1	
Ci. and Ci. St.	10000	-3.5	-4.0	+ 4.5	-5.5	+2.5	- 3.0	-6.5	+ 3.0	-1.0	+ 3.5	0.0	-3.0	-2.8	+35.4	
	9000	-3.5	-5.0	+ 4.5	-5.0	+2.5	- 8.0	-3.0	+ 8.0	-3.0	+11.0	-1.5	-1.0	-2.6	+35.0	
Ci. Cu.	8000	-3.0	-6.0	+ 3.5	-4.5	+2.0	- 9.0	-2.5	+11.0	-5.5	+13.5	-2.0	+1.0	-2.4	+34.6	
	7000	-1.5	-6.5	0.0	-4.5	-1.5	- 8.0	-6.5	+13.5	-5.0	+15.0	-2.0	+2.0	-2.2	+30.0	
A. St.	6000	0.0	-7.0	- 2.5	-4.5	-4.0	- 7.5	-9.0	+15.0	-2.0	+15.5	-0.5	+3.0	-2.0	+25.0	
	5000	0.0	-7.5	+ 2.5	-6.0	-2.5	- 8.0	-9.0	+17.0	0.0	+15.5	+1.5	+5.0	-1.8	+23.6	
A. Cu.	4000	0.0	-7.5	+ 8.5	-8.0	0.0	-10.0	-7.5	+20.0	0.0	+14.5	+2.5	+7.0	-1.6	+22.6	
	3000	0.0	-7.0	+10.0	-9.5	+2.0	-12.0	-3.5	+23.0	0.0	+13.0	+2.0	+7.5	-1.3	+21.0	
S. Cu.	2000	+1.0	-6.0	+ 7.5	-9.5	+2.0	-11.0	-1.0	+20.0	0.0	+11.0	0.0	+5.0	-1.0	+14.0	
Cu. and St.	1000	+3.5	-4.5	+ 5.0	-7.5	+1.5	- 7.0	0.0	+ 8.0	-1.5	+ 8.0	-1.5	+4.0	-0.8	+ 6.4	
Wind.	0000	+4.0	-2.5	+ 2.5	-4.0	+1.0	0.0	0.0	+ 6.0	-3.5	+ 3.0	-2.0	+3.0	-0.5	+ 1.3	
Means of the velocities, u_2, v_2		-0.3	-5.8	+ 4.2	-6.2	+0.5	- 7.6	-4.4	+13.1	-2.0	+11.2	-0.3	+3.8			
Radius, ϖ			1		3		5		1		3		5			
Product, ϖv_2			-5.8		-18.6		-38.0		+13.1		+33.6		+19.0			
+ u_2 = outward on radius.																+ u_1 = south. + v_1 = east.
+ v_2 = anticlockwise about center.																
I. $\varpi_I = 250,000$.																
II. $\varpi_{II} = 750,000$.																
III. $\varpi_{III} = 1,250,000$.																

TABLE 19.—Application of the formulæ for a cyclone. (Copy of Table 127, International Cloud Report.)

CONSTANTS AND FORMULÆ.

$\theta = 46^{\circ} 17'$
 $u = -1.25$
 $\varpi = 1,250,000 \text{ meters}$

$\lambda = 2n \cos \theta = .000100$
 $c = -\frac{2u}{\varpi} = .000002$
 $k = .000050$

$\frac{\lambda}{k-c} \frac{c}{2} = .000002$

$\psi = -\frac{c}{2} \varpi^2 z.$

$u = -\frac{c}{2} \varpi.$

$v = +\frac{\lambda}{k-c} \frac{c}{2} \varpi z.$

$w = +cz.$

$\cot i = \frac{\lambda}{k-c} z.$

DERIVED DISTANCES, VELOCITIES, CHECKS, AND INCLINATIONS.

ϖ	z	ϖz	u	v	w	ϖv	$\left\{ \begin{array}{l} \varpi w = \\ -2uz \end{array} \right\}$	$\cot i$	i	G	
<i>meters.</i>	<i>miles.</i>								$^{\circ}$	<i>mm.</i>	
1250000	777	1.00	1250000	-1.25	2.50	.0000020	3125000	2.5	2.00	26.6	0.34
1000000	621	1.56	1560000	-1.0	3.125	31	3125000	3.1	3.12	17.6	0.40
750000	466	2.78	2085000	-0.8	4.17	56	3125000	4.2	5.56	10.2	0.55
500000	311	6.25	3125000	-0.5	6.25	125	3125000	6.3	12.50	4.6	0.78
250000	155	25.00	6250000	-0.25	12.50	500	3125000	12.5	50.00	1.2	2.03
200000	124	39.06	7812000	-0.20	15.61	781	3125000	15.6	78.12	0.7	3.00
150000	93	69.40	10412000	-0.15	20.82	1388	3125000	20.8	138.80	0.4	5.37
100000	62	156.30	15630000	-0.10	31.26	3126	3125000	31.3	312.60	0.2	13.90
50000	31	625.00	31250000	-0.05	62.50	12500	3125000	62.5			
40000	25	977.00	39080000	-0.04	78.16	19540	3125000	78.2			
30000	19	1736.00	52080000	-0.03	104.16	34720	3125000	104.2			
20000	12	3906.00	78120000	-0.02	156.24	78120	3125000	156.2			
10000	6	15625.00	156250000	-0.005	312.50	.0312500	3125000	312.5			

was shown by the results of the observations that the movement at the cumulus level is much more rounded than in the lower strata, the difference being caused by the retardation of the air operating upon the surface irregularities of the ground. A congested or irregular inflow near the center will similarly increase the angle i , since the component u is increased and v diminished by it. The observations given on the weather maps do not record the conditions within 60 miles of the center with any definiteness. In hurricanes the core is about 30 miles broad and its boundary is quite sharp, which shows that the component v is highly developed, while u is small. But the hurricane also penetrates to much greater altitudes, as already mentioned. Finally, the gradient shows that there is a slow change near the outer limit, but that it increases very rapidly on approaching close to the center.

THE SPECIAL FEATURES OF THE CIRCULATION.

The special features of the circulation indicated on fig. 20, which may be mentioned are as follows: (1) It is evident that

(3) The approach of a moving particle to the axis of the cyclone is attended by an increase of the velocity of rotation, which accelerates rapidly as it passes into the upper strata. There it accomplishes the work of deflecting the eastward drift, and it expends some of its energy in that way. The result of this opposition to free motion is to spread out the top of the vortex, reduce its gyratory velocity, and change the relations of v_2 and u_2 . In the undisturbed gyratory motion the component v_2 becomes very great in comparison with u_2 , and u_2 is always a small quantity. An inspection of Table 18 shows, however, that in the strata, between 5,000 and 10,000 meters the radial velocity u_2 is relatively large, and the angle of the inclination instead of being nearly 0° is from 25° to 35° on the inner circle II. This means that the original coefficient upon which

the dimensions of the cyclone depend, namely, $c = -\frac{2u}{\omega}$, does

not remain a constant, but increases from the boundary toward the axis of the gyration. Thus we obtain,

FIG. 20.—General scheme of the structure of cyclones.

this scheme avoids entirely the primary difficulties attending the Ferrel and also the German types of circulation. Each of these divided the cyclone into two parts having special properties. Ferrel divided his cyclone at a vanishing rotational velocity, $v = 0$, which involved a circulation in opposite directions on either side of it; the German type consists of two parts, separated by a discontinuous movement at the circle of the maximum velocity for v . The pure vortex, on the other hand, has only one law to deal with, and that, too, the simplest of all, in accordance with which the motion is generated. (2) Neither of the other types provides for a true calm region at the center of the cyclone, commonly observed in hurricanes as the eye of the storm. Ferrel's formula 402b shows that v increases from the circle $R = 0.707$ to the very center; the formulae 450 and 471 show that the velocity v decreases gradually from the circle bounding the inner region toward the center, but does not vanish till reaching it. The construction here proposed indicates that since v is a function of the height z as well as of the radial distance ω , the air in streaming toward the center is gradually lifted above the ground by purely dynamic action and leaves a core without gyratory circulation.

$$\begin{aligned} c_1 &= .0000352 = \frac{0.6}{1250000} \\ c_{II} &= .0000053 = \frac{4.0}{750000} \\ c_{III} &= .0000005 = \frac{8.8}{250000} \end{aligned}$$

This involves an expenditure of energy in the struggle attendant upon intruding into the swiftly moving upper stratum. Furthermore, as was previously pointed out, Ferrel's theory of the slowing down of the excessive eastward velocities which would arise from the pure vortex law of the conservation of areas applied to the general cyclone, is this, friction is largely concerned with the operation. It seems however, that a much more efficient cause of retardation at the interaction between these two types of motion, namely, the linear and the rotary, by which the lower strata thrust themselves into the higher. The effect is to enlarge the size of the vortex tube at the top by the resistance, deflect the eastward drift into sinuous curves, slow down the eastward velocity, and thus restrain the general cyclonic movement from

TABLE 20.—*Dimensions and velocities in the waterspout off Cottage City, Vineyard Sound, Mass., August 19, 1896. (Copy of Table 128 International Cloud Report.)*

Working formulæ: $v^2=2gz$; $u=\frac{v}{z}$; $w=\frac{2v}{\omega}$; $\omega v=\text{Const.}$; $2zu=\omega w$; $\frac{c}{z}=\frac{4g}{v\omega}$.												
Dimensions in meters.			Velocities in meters per second.				Dimensions in feet.			Velocity in miles per hour.		
h	z	ω	u	v	w	$2zu=\omega w$	h	z	ω	u	v	w
1280	0	∞	—	0	0	0	4200	0	∞	—	0	0
1278	2	518.4	3.13	6.26	0.02	12.53	4193	7	1701	7.0	14.1	0.04
1189	91	76.9	0.46	42.24	1.40	84.49	3901	299	253	1.0	94.4	2.5
1097	183	60.8	0.29	53.39	1.76	106.79	3599	601	200	0.6	119.5	3.9
1006	274	44.3	0.27	73.31	3.31	146.61	3301	899	145	0.6	164.	7.4
914	366	38.3	0.23	84.72	4.42	169.45	2999	1201	125	0.5	189.	9.9
731	549	31.3	0.19	103.77	6.63	207.53	2398	1802	102	0.4	233.	14.9
549	731	27.1	0.16	119.73	8.83	239.47	1802	2308	89	0.4	268.	19.8
457	823	25.5	0.15	127.04	9.94	254.10	1409	2701	84	0.3	284.	22.2
366	914	24.3	0.15	133.89	11.04	267.78	1201	2999	79	0.3	300.	24.7
183	1097	22.1	0.13	146.68	13.25	293.36	601	3599	72	0.3	328.	29.6
146	1134	21.8	0.13	149.13	13.70	298.26	479	3721	72	0.3	333.	29.7
0	1280	20.5	0.13	158.44	15.46	316.89	0	4200	67	0.3	354.	34.6

$$c = 0.01208$$

excessive values. (4) The resultant of these component forces and velocities is to produce a circulation along the parallels of latitude which may be represented by the upper part of fig. 20. The upper clouds of the cirrus region precede the cyclone proper as forerunners of this type of circulation; the lower clouds follow in succession, till precipitation is produced at the elevation of 1,000 to 3,000 meters; in very rapid circulations the eye of the storm is fully developed; the clearing up is more abrupt on the westward than on the eastward side of the cyclone. (5) The progressive movement of storms is partly an effect of the cyclone covering an area of sufficient extent to be in different latitudes, so that variations in $\cos \theta$ amount to something. But other circumstances are more important. By Table 33, Section IV, International Cloud Report, it is seen that the northward components for the group of areas which are covered by the currents of air from the south are greater than those from the north. That is to say, the movement in the streams from the Tropics is more rapid than that from the polar regions, and the result is to roll up the eastward side of the cyclone to the north more than the westward to the south. The cyclone tends to rotate along the front of a high area toward the north. At the same time its top is fastened by means of the circulation into the eastward drift at 4,000 meters elevation, and these two components make the storm move northeastward in the central and eastern portions of the United States. If the general cyclone has other directions, as when hurricanes form in the Caribbean Sea, the same principles hold, and the storm there first moves westward, then northward and northeastward, because the general circulation is controlled in that region by the anticyclone in the southern portions of the north Atlantic Ocean. (6) It has been shown that the currents which feed the cyclone have different velocities at different altitudes, being greatest from 2,000 to 5,000 meters above the ground. Each stratum forms a stream for itself conforming to the general law, but modifies its dimensions according to the constants pertaining to the special locality. Since these different strata have thus distinct local movements, especially considering the variable temperatures and densities of the currents from the north and south, respectively, it fol-

lows that the conditions are favorable for the formation of turbulent minor circulations of all kinds. The movement of the air is therefore partly congested, and partly runs in free whirls, the difference of equilibrium in temperature being gradually reduced to the proper normal value for the latitude and altitude by this forced intermingling of the subordinate parts of the cyclone. This process of restoring to an equilibrium the temperature of masses, bearing with them that of the region from which they came, is generally completed by the time the 5,000-meter level is reached, judging from the records of the balloon observations. In summer, when the eastward drift is relatively slow, the pure vertical convective ascension may extend up to 10,000 meters. This is much more likely to happen on the eastward than on the westward side of a cyclone, because the vortex components throw back the eastward movement upon itself, and thus make the strata more stagnant in vertical directions. It has also been found practically very difficult to make the kites fly on the east of the low center, the best ascensions being made on the southerly and westerly quadrants. (7) The entire problem of analyzing the movements of the air in their details is so exceedingly complicated that only slow improvements in dynamic meteorology can be expected. A clearer idea of the fundamental conditions may, however, enable us to advance more rapidly than is now anticipated. It will be a very important gain if meteorology can free itself from some of the theories which have so long prevailed, but which now are seen to be quite untenable, and have seriously retarded its advancement.

THE VELOCITIES IN TORNADES.

The motions in tornadoes are similar to those in cyclones, yet the tube is not only inverted in position, but the stream lines occupy only the central portions, and the lines of ϕ become tangent to the plane whose height is H_0 at certain distances from the center. The fundamental equations for the tornado are

303-308. $\phi = + \frac{c}{2} \omega^2 z$ holds for the tornado with vertical axis positive downward, and the single bounding plane at the dis-

tance H_0 above the ground. $\psi = -\frac{c}{2}\omega^2 z$ is the equation for the cyclone with the vertical axis positive upward. A multitude of minor relations and comparisons can be drawn from the two sets of equations 303-308 and 488-490. If the lower parts of fig. 20 be looked at as if the horizontal axis were the vertical, we have a picture of the half of a tornado tube. The diagram is not good because not drawn to scale, but the idea is easily understood. Hence, one law serves for all types of local storms, cyclones, hurricanes, and tornadoes, which thus theoretically differ from each other only in their dimensions and in the details by which they are formed. Cyclones are generated chiefly by horizontal convection currents; hurricanes have a stronger vertical convection current and also horizontal convection currents; tornadoes arise chiefly from vertical con-

vection, assisted by some horizontal currents which counter flow in the cumulus level.

It may be remarked that the stream lines indicated by Ferrel, page 300, *Recent Advances*, are conjectural only, and do not conform to the theory of stream lines in a vertical vortex tube, nor to observation, which shows that the air is quiet close up to the boundary of the tornado tube.

THE WATERSPOUT OFF COTTAGE CITY, MASS., AUGUST 19, 1896.

The result of the computation on this interesting waterspout is added, and it shows the dimensions and velocities in metric and English measures which were derived from the observed distances and the formulæ. The most important feature is the value of the vertical velocity of 35 miles per hour at the sea level. See Table 20.

VI.—CERTAIN MATHEMATICAL FORMULÆ USEFUL IN METEOROLOGICAL DISCUSSIONS.¹

THE NEED OF A STANDARD SYSTEM OF FORMULÆ.

There are a large number of mathematical papers that have been written by meteorologists in the exposition of various theories, which must be thoroughly considered by students who seek to go beyond a descriptive statement of the problems into a close examination of the principles upon which the solutions rest. The question arose at an early stage in my study of comparative meteorology as to the form in which such mathematical discussions should be presented to the public. To traverse the entire range of treatises and explain them in detail was clearly impracticable; to adopt an abstract mathematical synopsis, such as is found in Carr's or Laska's synopsis of pure mathematics, was to put too great a strain upon readers who are not specialists in mathematical meteorology. Finally it seemed to me to be a fair compromise to take the following course: (1) reduce the important papers to one common standard notation, and (2) make an analysis of the result in a sufficiently expanded form to enable a good reader to follow the series of equations without difficulty. The only step required to transform the contents of the mathematical compendium as given in chapters 10 and 11 of the International Cloud Report into a complete treatise on analytic meteorology is to supply such transition precepts as are usually placed between the formulæ to aid the thought. It is, however, a distinct advantage for a working use of the formulæ, to one who has once become familiar with such problems, to dispense with these explanatory sentences, which only take up space. A ready reference to the standard equations under each subject is quickly appreciated by any one who uses these formulæ in a practical way, just as one would use a mathematical table in computing. It is my purpose to complete such a collection of formulæ, in addition to the tables contained in my report on Eclipse Meteorology and Allied Problems, Weather Bureau Bulletin I, 1902, by appropriate tables covering the subjects, spherical harmonics, thermodynamics, and the kinetic theory of gases, because these are indispensable in meteorological studies. I have taken the opportunity in this connection to present several original sets of formulæ, which have an advantage in their applications to meteorological problems, and it is my purpose to call attention to some of them in this paper.

THE GENERAL EQUATIONS OF MOTION.

The methods of deriving the general equations of motion on the rotating earth, as presented in Ferrel's paper, "The motions of fluids and solids on the earth's surface," or in the standard treatises of hydrodynamics, are so complicated as to discourage all who are not expert mathematicians from an examination of the solution. The fact that Ferrel did not evaluate the total differential of inertia $\frac{d(u, v, w)}{dt}$, introduced an error into the equations contained in his "Mechanics and general motions of the atmosphere," United States Coast Survey Report, 1875, Appendix 20; this was eliminated in his "Recent advances in meteorology," Annual Report of the Chief Signal Officer, 1885, Appendix 71. There are no doubt many ways of solving this problem, but the following is original, as expanded from Table 75, International Cloud Report, and it leaves little to be desired in respect of simplicity and completeness.

(1) THE POLAR EQUATIONS OF MOTION ON THE ROTATING EARTH.

Using the notation already adopted in Paper II of this

series,² we write the primary equations of acceleration of motion referred to axes which have their origin at the center of a nonrotating earth, as follows:

The accelerations due to motion and to external forces are,

$$\begin{aligned} 155. \quad & -\frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{\partial V}{\partial x} = \frac{du}{dt} - v\omega_3 + w\omega_2 \\ & -\frac{1}{\rho} \frac{\partial P}{\partial y} - \frac{\partial V}{\partial y} = \frac{dv}{dt} - w\omega_1 + u\omega_3 \\ & -\frac{1}{\rho} \frac{\partial P}{\partial z} - \frac{\partial V}{\partial z} = \frac{dw}{dt} - u\omega_2 + v\omega_1 \end{aligned}$$

where the angular velocities of motion for a point are

$$166. \quad \omega_1 = -\frac{v}{r} \quad \omega_2 = +\frac{u}{r} \quad \omega_3 = +\frac{v}{r \tan \theta}.$$

Compare diagram in my Report, page 498, or Basset, pages 13 and 14, noting the transformations of notation.

In case the earth rotates with the constant angular velocity n , carrying the fixed axes with it, the linear velocities (u, v, w) and the angular velocities ($\omega_1, \omega_2, \omega_3$) are changed as follows, denoting these terms on the rotating earth with primes:

$$\begin{aligned} 177. \quad & u' = u \\ & v' = v + n r \sin \theta \\ & w' = w \\ 178. \quad & \omega'_1 = -\frac{v + n r \sin \theta}{r} \\ & \omega'_2 = +\frac{u}{r} \\ & \omega'_3 = +\frac{v + n r \sin \theta}{r \tan \theta}. \end{aligned}$$

This is due to the fact that the rotation of the earth adds the velocity $n r \sin \theta = n \varpi$ to the eastward linear velocity, because ϖ is the perpendicular distance from the axis of rotation.

The differentials $\frac{du'}{dt}$, $\frac{dv'}{dt}$, $\frac{dw'}{dt}$ evaluate into,

$$\begin{aligned} 179. \quad & \frac{du'}{dt} = \frac{du}{dt} \\ & \frac{dv'}{dt} = \frac{dv}{dt} + \frac{d(n r \sin \theta)}{dt} = \frac{dv}{dt} + u n \cos \theta + w n \sin \theta \\ & \frac{dw'}{dt} = \frac{dw}{dt} \end{aligned}$$

since $u = \frac{rd\theta}{dt}$ and $w = \frac{dr}{dt}$ by formulæ 153, page 497, of the International Cloud Report.

Substituting these values in the equations of motion for the rotating earth, which are the same as those of 155 with the letters all primed, and taking the equivalents of dx, dy, dz in polar coordinates from 153, we have:

$$\begin{aligned} 180. \quad & -\frac{1}{\rho} \frac{\partial P}{r \partial \theta} = \frac{du}{dt} - (v + n r \sin \theta) \frac{(v + n r \sin \theta)}{r \tan \theta} + w \frac{u}{r}, \\ & -\frac{1}{\rho} \frac{\partial P}{r \sin \theta \partial \lambda} = \frac{dv}{dt} + w \frac{(v + n r \sin \theta)}{r} + u \frac{(v + n r \sin \theta)}{r \tan \theta} \\ & \quad + u n \cos \theta + w n \sin \theta, \\ & -\frac{1}{\rho} \frac{\partial P}{\partial r} - g = \frac{dw}{dt} - u \frac{u}{r} - (v + n r \sin \theta) \frac{(v + n r \sin \theta)}{r}. \end{aligned}$$

The external forces derived from the potential V are:

$$-\frac{\partial V}{\partial x} = 0, \quad -\frac{\partial V}{\partial y} = 0, \quad -\frac{\partial V}{\partial z} = -g.$$

¹ Reprinted from the Monthly Weather Review for June, 1902.

² See Monthly Weather Review for February, 1902, Vol. XXX, p. 81.

Performing the algebraic work, these equations reduce to

$$\begin{aligned}
 181. \quad & -\frac{1}{\rho} \frac{\partial P}{r \partial \theta} = \frac{du}{dt} - \frac{v^2 \cot \theta + uv}{r} \\
 & \quad - 2n \cos \theta \cdot v - r n^2 \sin \theta \cos \theta, \\
 & -\frac{1}{\rho} \frac{\partial P}{r \sin \theta \partial \lambda} = \frac{dv}{dt} + \frac{uv \cot \theta + vw}{r} \\
 & \quad + 2n \cos \theta \cdot u + 2n \sin \theta \cdot w, \\
 & -\frac{1}{\rho} \frac{\partial P}{\partial r} - g = \frac{dw}{dt} - \frac{u^2 + v^2}{r} - 2n \sin \theta \cdot v - r n^2 \sin^2 \theta.
 \end{aligned}$$

The successive terms are the inertia, the centrifugal forces, the deflecting force, and the forces which change the figure of the earth from a sphere into an ellipsoid of revolution.

(2) THE CYLINDRICAL EQUATIONS OF MOTION ON THE ROTATING EARTH.

If the axis of rotation of the earth is taken as the axis of rotation in cylindrical coordinates, the tangential velocity $= v + n \varpi$; but if the axis of rotation is any radius of the earth extended above the surface, the tangential velocity becomes $= v + n \varpi \cos \theta$. Hence we have, in cylindrical coordinates,

$$\begin{aligned}
 182. \quad & u' = u & w'_1 &= 0 \\
 & v' = v + n \varpi \cos \theta & w'_2 &= 0 \\
 & w' = w & w'_3 &= n \cos \theta + \frac{v}{\varpi}.
 \end{aligned}$$

The differentials $\frac{du'}{dt}$, $\frac{dv'}{dt}$, $\frac{dw'}{dt}$ evaluate into,

$$\begin{aligned}
 183. \quad & \frac{du'}{dt} = \frac{du}{dt} \\
 & \frac{dv'}{dt} = \frac{dv}{dt} + \frac{d(n \varpi \cos \theta)}{dt} = \frac{dv}{dt} + u n \cos \theta \\
 & \frac{dw'}{dt} = \frac{dw}{dt}
 \end{aligned}$$

since $u = \frac{d\varpi}{dt}$, by formulæ 152, and $\cos \theta$ is a constant. Sub-

stituting these values in the equations of motion for the rotating earth, which are the same as those of 155, with the letters all primed, and taking the equivalents of dx , dy , dz , in cylindrical coordinates from 152, we have:

$$\begin{aligned}
 184. \quad & -\frac{1}{\rho} \frac{\partial P}{\partial \varpi} = \frac{du}{dt} - (v + n \varpi \cos \theta) \left(n \cos \theta + \frac{v}{\varpi} \right) \\
 & -\frac{1}{\rho} \frac{\partial P}{\varpi \partial \varphi} = \frac{dv}{dt} + u n \cos \theta + u \left(n \cos \theta + \frac{v}{\varpi} \right) \\
 & -\frac{1}{\rho} \frac{\partial P}{\partial z} - g = \frac{dw}{dt}.
 \end{aligned}$$

The external forces derived from the potential V are:

$$-\frac{\partial V}{\partial x} = 0, \quad -\frac{\partial V}{\partial y} = 0, \quad -\frac{\partial V}{\partial z} = -g.$$

Performing the algebraic work, these equations reduce to

$$\begin{aligned}
 185. \quad & -\frac{1}{\rho} \frac{\partial P}{\partial \varpi} = \frac{du}{dt} - 2n \cos \theta \cdot v - \frac{v^2}{\varpi} \\
 & \quad = \frac{du}{dt} - (2n \cos \theta + \nu_1) v \\
 & -\frac{1}{\rho} \frac{\partial P}{\varpi \partial \varphi} = \frac{dv}{dt} + 2n \cos \theta \cdot u + \frac{uv}{\varpi} \\
 & \quad = \frac{dv}{dt} + (2n \cos \theta + \nu_1) u \\
 & -\frac{1}{\rho} \frac{\partial P}{\partial r} - g = \frac{dw}{dt}
 \end{aligned}$$

where the term $+ \varpi n^2 \cos^2 \theta$ is neglected in the first equation, and $\nu_1 = \frac{v}{\varpi}$ the relative angular velocity.

The successive terms are the inertia, the deflecting force, and the centrifugal forces.

REMARKS ON THE SEVERAL TERMS IN THE GENERAL EQUATIONS OF MOTION.

It is customary to add to the terms developed in a frictionless medium, a term expressing the retardation of acceleration due to friction, either in Ferrel's form $+ k(u, v, w)$, which is proportional to the velocity and expresses a sliding friction, or in Oberbeck's form $\frac{k}{\rho} \Delta^2(u, v, w)$, which expresses a retardation proportional to the turbulent internal resistances of a mixing fluid. This function is hard to evaluate on account of the uncertainty which attaches to the invisible internal motions, and to the effect of discontinuous surfaces separating different velocities and temperatures. Near the ground turbulent motions and large coefficients of friction up to about 300–500 meters are required; above this level and especially in the higher strata the coefficient of friction is very small.

The inertia terms $\frac{d(u, v, w)}{dt}$ disappear in steady motion, and they are small in slow changes of velocities. There are, however, cases in which inertia may amount to a considerable quantity, as where a tornado, in passing along its path, sucks in new masses of air, and transforms them suddenly from rest into excessively rapid motion. Also, when the cyclonic vortex raises masses of air from strata having slow motion into strata of rapid velocities; but especially where countercurrents meet, and the stream lines are bent and reflexed in their direction.

These two terms, friction and inertia, act in the path of motion and they directly affect the quantity of kinetic energy possessed by the elementary masses. All forces which act at right angles to the path, such as the centrifugal and the deflecting forces, do not change the momentum, but they do alter the direction of the path. Hence, in integrating for the kinetic energy in an orbit, or in a circuit, the centrifugal and the deflecting forces drop out of the equations, but they must be retained when discussing the angle that the stream line makes with the isobars, which angle expresses the influence of the velocity potential function on the motion. The following integration of the general equations will establish these propositions.

INTEGRATION OF THE GENERAL EQUATIONS OF MOTION IN POLAR COORDINATES.

Make the following substitutions in 181:

$$\begin{aligned}
 197. \quad & \frac{v}{r} = \nu \sin \theta \\
 & \frac{v^2 \cot \theta}{r} = v \cos \theta \cdot \nu \\
 & \frac{u v \cot \theta}{r} = u \cos \theta \cdot \nu
 \end{aligned}$$

and neglect the terms in n^2 , which are very small, with the result that,

$$\begin{aligned}
 200. \quad & -\frac{1}{\rho} \frac{\partial P}{\partial x} = \frac{du}{dt} - \cos \theta (2n + \nu) v + \frac{uv}{r} \\
 & -\frac{1}{\rho} \frac{\partial P}{\partial y} = \frac{dv}{dt} + \cos \theta (2n + \nu) u + \sin \theta (2n + \nu) v \\
 & -\frac{1}{\rho} \frac{\partial P}{\partial z} = \frac{dw}{dt} - \sin \theta (2n + \nu) v - \frac{u^2}{r} + g.
 \end{aligned}$$

Now multiply these equations respectively by dx , dy , dz , and remember that $v \partial x = u \partial y$, $u \partial x = u \partial z$, $w \partial y = v \partial z$; take the sum of the partial differentials, the result being,

because B at the top of the homogeneous column l is negligible compared with B_i at the bottom of it. B_i is in this connection the barometric pressure at the surface, and $B_i = B_n = 0.760$ meter.

II. We have by 50, page 490, for the standard weight of the atmosphere,

$$507. \quad p_o = \sigma h = \sigma_o l = \sigma_m B_n. \quad \text{Hence,}$$

$$508. \quad h = \frac{\sigma_o}{\sigma} l = \frac{\sigma_m}{\sigma} B_n. \quad \text{That is, } h = l \text{ for } \sigma = \sigma_o. \quad \text{Hence,}$$

$$509. \quad h = l = \frac{\sigma_m}{\sigma_o} B_n. \quad \text{Therefore,}$$

$$510. \quad s = \frac{\sigma_m}{\sigma_o} = \frac{l}{B_n} = \frac{RT}{B_n} = \frac{13,595.8}{1.29305} = \frac{7,991.04}{0.760} = 10,514.5.$$

$$511. \quad dh = \frac{\sigma_m}{\sigma_o} dB = \frac{RT}{B_n} dB = 10,514.5 dB = s dB.$$

$$512. \quad \frac{dh}{dx} = 10,514.5 \frac{B_o - B}{D} = 10,514.5 \frac{G}{111 \ 111} = 0.09463 G.$$

III. Let I' = the gradient force per meter; that is, for $dx = 1$.

$$513. \quad I' = \Delta P = g_o \rho_m \Delta B \text{ in terms of the units of force } P.$$

$$514. \quad I' = \Delta p = \sigma_m \Delta B \text{ in terms of the units of weight } p.$$

The gradient force changes with the temperature.

Let I'_o = the gradient force for $T_o = 273^\circ \text{C.}$ and $B_n = 0.760$ meter.

I' = the gradient force for T and B .

$$515. \quad I' = I'_o \frac{T}{T_o} \frac{B_n}{B}.$$

IV. To evaluate $-\frac{1}{\rho} \frac{\partial P}{\partial x}$ and $-\frac{1}{\rho} \frac{\partial p}{\partial x}$:

516. We have $P_o = g_o \rho_m B_n$; and hence,

$$517. \quad -\frac{1}{\rho} \frac{\partial P}{\partial x} = -\frac{1}{\rho} g_o \rho_m \frac{\partial B}{\partial x} = -\frac{1}{\rho} \frac{g_o \rho_m}{111 \ 111} G = -\frac{0.0012 G}{\rho}.$$

(G is in meters.)

518. Also, we have $p_o = \sigma_m B_n$; and hence,

$$519. \quad -\frac{1}{\rho} \frac{\partial p}{\partial x} = -\frac{1}{\rho} \sigma_m \frac{\partial B}{\partial x} = -\frac{1}{\rho} \frac{\sigma_m}{111 \ 111} G = -\frac{0.12237 G}{\rho}.$$

(G is in meters.)

Numerous other evaluations of $-\frac{1}{\rho} \frac{\partial P}{\partial x}$ are given in Table 65, p. 489, of the Cloud Report.

EVALUATION OF THE GRADIENTS IN POLAR COORDINATES.

Since the angular velocity of the rotating earth is $n \sin \theta = \frac{v'}{r}$, where v' is the absolute eastward velocity, and $r = 6,370,191 + h$ meters, we have $n = 0.00007292$, and also $n \cos \theta = \frac{v' \cot \theta}{r}$, in which r can be taken practically equal to R . The general polar equations of motion become, by substituting these values in 181,

$$194. \quad \begin{aligned} -\frac{1}{\rho} \frac{\partial P}{\partial x} &= \frac{du}{dt} - \frac{\cot \theta}{r} (2v' + v)v + \frac{uv}{r} \\ -\frac{1}{\rho} \frac{\partial P}{\partial y} &= \frac{dv}{dt} + \frac{\cot \theta}{r} (2v' + v)u + (2v' + v) \frac{v}{r} \\ -\frac{1}{\rho} \frac{\partial P}{\partial z} &= \frac{dw}{dt} - \frac{1}{r} (2v' + v)v - \frac{u^2}{r} + g. \end{aligned}$$

The terms in n^2 which give the figure to the rotating earth have been omitted, and the inertia terms become equal to zero for steady motions of the atmosphere; also, for all except computations of great precision the terms in w can be neglected.

To evaluate the acceleration $\frac{1}{\rho} \frac{\partial P}{\partial x}$, we have, first, from 47,

$$47a. \quad \frac{1}{\rho} = \frac{1}{\rho_o} \frac{P_o}{P} \frac{T}{T_o} = \frac{1}{\rho} \frac{P_o}{P} (1 + \alpha t) \\ = \frac{1}{\rho_o} \frac{B_n}{B} \frac{T}{T_o} \frac{1}{n_1}, \text{ for variations of gravity, since } g = g_o n_1, \\ = \frac{1}{\rho_o} \frac{760}{273} \frac{T}{B} \text{ for constant gravity and } B \text{ in mm.}$$

From the formulae on page 489,

$$47b. \quad \frac{1}{\rho} \frac{\partial P}{\partial x} = \frac{1}{\rho} g_o \rho_m \frac{dB}{dx}, \text{ and since } \frac{dB}{dx} = \frac{G_x}{D} = \frac{G_x}{111 \ 111} \text{ in meters} \\ = \frac{1}{\rho} \frac{g_o \rho_m}{111 \ 111} G_x \text{ for the gradient measured in meters} \\ = \frac{1}{\rho} \frac{g_o \rho_m}{111 \ 111} G_x \text{ for the gradient } G_x = B_o - B \text{ in mm} \\ = \frac{1}{\rho_o} \frac{B_o}{T_o} \frac{T}{B} \frac{g_o \rho_m}{D} G_x$$

$$520. \quad = \frac{13,595.8}{0.00129305} \times \frac{760}{273} \times \frac{9.806}{111 \ 111} \times \frac{T}{B} \times G_x$$

$$521. \quad = 0.0025833 \frac{T}{B} G_x$$

$$522. \quad = \frac{\cot \theta}{r} (2v' + v)v, \text{ by equation 194. Hence,}$$

$$523. \quad G_x = + 387.102 \frac{B}{T} \frac{\cot \theta}{r} (2v' + v)v,$$

and similarly,

$$G_y = - 387.102 \frac{B}{T} \frac{\cot \theta}{r} (2v' + v)u,$$

$$G_z = + 387.102 \frac{B}{T} \left[\frac{1}{r} (2v' + v)v + \frac{u^2}{r} - g \right].$$

Since v' is a function of θ , that is, $v' = n r \sin \theta$, these terms can be computed by simple tables, such as those in Tables 104, 105, 106, of the International Cloud Report, where some of the terms are evaluated. By expressing the variation of $387.102 \times \frac{B}{T}$ in a table with B and T as the arguments, the several products can be quickly computed.

Examples:

$$\text{I. For } B = 700 \text{ mm. and } T = 260^\circ \text{C., } \frac{B}{T} = 2.6923;$$

For $\theta = 30^\circ$ north polar distance, $2v' = 464.5$ meters per second;

For $v = 40$ meters per second, $(2v' + v)v = 20,180$.

Hence, $G_x = 387.102 \frac{B}{T} \frac{\cot \theta}{R} (2v' + v)v = 5.71$ millimeters per 111 111 meters.

II. This latter has been computed from the tables as follows:

$$387.102 \frac{B}{T} = 1,042.2;$$

$$\frac{v'}{R} \cot \theta \cdot 2v = 0.005052, \text{ by Table 104;}$$

$$\frac{\cot \theta}{R} \cdot v \cdot v = 0.000435, \text{ by Table 106.}$$

The sum of these is $\frac{\cot \theta}{R} (2v' + v)v = 0.005487$. Hence

the product, $G_x = 1,042.2 \times 0.005487 = 5.71$ millimeters per 111 111 meters. Similarly, the gradients G_y and G_z can be computed.

For these values of B and T , we find in other examples,

$$\left. \begin{array}{l} \theta = 40^\circ \\ v = 45^\circ \end{array} \right\} G_x = 6.89 \quad \left. \begin{array}{l} \theta = 50^\circ \\ v = 50^\circ \end{array} \right\} G_x = 5.23 \quad \left. \begin{array}{l} \theta = 60^\circ \\ v = 55^\circ \end{array} \right\} G_x = 4.47.$$

We are at last in a position to examine the system of gradients in the United States on the 10,000-foot plane and on the 3,500-foot plane. For we have obtained by the nephoscope and theodolite observations as given in the Cloud Report a large number of corresponding values of u and v , which enter these equations. The values of B and T on these planes have been carefully determined for each month, and also the gradients by which such values can be determined at any time. This will enable us to discuss the effect of friction at these planes, by means of the residuals which occur between the values of G as found by these formulæ and those read off from the charts of isobars contained in the Barometry Report of 1900-1901.

Furthermore, our Weather Bureau stations will soon be provided with suitable tables for computing pressures on the 3,500-foot and the 10,000-foot planes, and this will give daily configurations of isobars on these two levels. If, in addition, we had measures of the velocity of the clouds, $q(u, v, w)$, above each station by means of nephoscopic observations, it would enable us to make complete dynamic computations of the forces acting in cyclones and anticyclones, as is seen by an inspection of the formulæ.

Since the tabular computations are constructed for average conditions, it is of the utmost importance that *check observations* be made on those two planes in order to control these dynamic discussions and make them more perfect. Such observations can be made by balloon ascensions up to 2 or 3 miles, or by kite ascensions up to 10,000 feet, or by certain computations on cloud observations.

It seems to me very clear that a series of suitable research explorations would soon result in placing our dynamic meteorology upon a satisfactory scientific basis, and put an end to the fruitless speculations which have done so little to advance our knowledge of the laws of the atmospheric motions.

THE EQUATION OF CONTINUITY, AND SOME DERIVED RELATIONS.

1. The equation of continuity can be found as follows:

Consider a cylinder of the height z and radius ϖ , into which air of the density ρ streams equally from all sides with the velocity $-u$, since the direction is negative.

FIG. 22.—Illustrating the formation of the equation of continuity.

The amount of instreaming air in the unit of time is

$$-2\pi\varpi z u\rho.$$

If the air is incompressible, then there will stream into a cylinder, whose radius is smaller by $d\varpi$, the amount $-2\pi(\varpi - d\varpi)z u\rho$, and at the same time there will escape upward between these two cylinders the amount $2\pi\varpi d\varpi w\rho$. Hence

$$524. \quad -2\pi\varpi z u\rho + 2\pi(\varpi - d\varpi)z u\rho = -2\pi d\varpi z u\rho = 2\pi\varpi d\varpi w\rho.$$

Integrating along the entire radius from 0 to ϖ , we have,

$$-2\pi\rho \int_0^\varpi z u d\varpi = 2\pi\rho \int_0^\varpi \varpi d\varpi w.$$

Therefore, $-zu\varpi = \frac{1}{2}\varpi^2 w$, and the equation of continuity becomes

487.

$$-2uz = w.$$

This applies to pure vortex motion, and it finds some examples in the atmosphere, such as in tornadoes, in many hurricanes, and in some highly developed cyclones.

It may be remarked that in treating of the general circulation of the atmosphere, the application of the pure vortex law $\varpi v = \text{constant}$, has failed to give correct results, for example in the writings of Ferrel, von Helmholtz, Oberbeck, Sprung, and others. This leads to the theory of contracting rings on the earth with progressive motion towards the poles, or expanding rings with progression towards the equator. While the law of the sum of the momenta $\sum mv = 0$ must prevail, the rings are in nature broken up into such complex stream lines as to render integration by the simple vortex law too rough and ready a method. We must, therefore, study the theory of typical stream lines, before attempting any general integration for the entire circulation.

The following derived relations are convenient:

$$2. \text{ Since } -2uz = w, \text{ we have } -u = \frac{w}{2z} = \frac{c}{2} \varpi, \text{ if } w = cz.$$

$$= \frac{c}{2} \varpi, \text{ if } w = cz.$$

$$3. \text{ For } z \text{ and } -u \text{ both constant, we have } \varpi z u = -\text{const.} = \psi.$$

Hence, $z = -\frac{\text{const.}}{u\varpi} = -\left(\frac{\text{const.}}{c/2}\right)\frac{1}{\varpi^2}$, and by differentiation,

$$dz = +\frac{\text{const.}}{c/2} \cdot \frac{2\varpi d\varpi}{\varpi^4} = -\varpi^2 z \frac{2d\varpi}{\varpi^3} = -2z \frac{d\varpi}{\varpi}. \text{ Therefore,}$$

$$\frac{dz}{z} = -2 \frac{d\varpi}{\varpi}; \text{ also, } \varpi \frac{dz}{dt} = -2z \frac{d\varpi}{dt}.$$

4. These give the form for the current function ψ , and the velocity potential φ , in two cases.

$$488. \quad \text{I. } \psi = u\varpi z = -\frac{c}{2}\varpi^2 z = \varphi z. \quad \text{II. } \psi = -cz.$$

$$489. \quad \varphi = -\frac{c}{2}\varpi^2. \quad \varphi = -c.$$

$$a = \frac{\lambda}{k-c}. \quad a = \frac{\lambda}{k}.$$

5. If the current function is modified through a deflecting force and also through friction, then the equation of motion has two solutions, so that the roots of

$$u \frac{\partial v}{\partial \varpi} + \frac{uv}{\varpi} + \lambda u + kv = 0$$

are, by 438,

$$1. \quad u = -\frac{c}{2}\varpi.$$

$$v = +\frac{\lambda}{k-c} \frac{c}{2}\varpi = -\frac{\lambda}{k-c} u.$$

$$2. \quad u = -\frac{c}{\varpi}.$$

$$v = +\frac{\lambda}{k} \frac{c}{2} = -\frac{\lambda}{k} u.$$

In obtaining the velocities of the rotation, v , we can modify the current function, as follows, namely, multiply by

$$a = \frac{\lambda}{k-c}, \text{ and } a = \frac{\lambda}{k} \text{ in the two cases.}$$

6. Hence, by using Stokes's current functions, we find for the velocities u, v, w in the two cases,

490. Case I.
$$u = + \frac{1}{\omega} \frac{\partial \psi}{\partial z} = \frac{\varphi}{\omega} = - \frac{c}{2} \omega.$$
$$v = + \frac{a\psi}{\omega} = - \frac{\lambda z}{k-c} \frac{c}{2} \omega = + \frac{\lambda}{k-c} z u.$$
$$w = - \frac{1}{\omega} \frac{\partial \psi}{\partial \omega} = + cz = - \frac{2u}{\omega} z = - \frac{2v}{\omega} \frac{k-c}{\lambda}.$$

Case II.
$$u = + \frac{1}{\omega} \frac{\partial \psi}{\partial z} = \frac{\varphi}{\omega} = - \frac{c}{\omega}.$$
$$v = + \frac{a\psi}{\omega} = - \frac{\lambda}{k} \frac{c}{\omega} z.$$
$$w = - \frac{1}{\omega} \frac{\partial \psi}{\partial \omega} = 0.$$

7. In unconstrained motion the vortex law of preservation of areas is

$$v\omega = - \frac{c}{2} \omega^2 z = 2g \frac{\omega z}{v} = \omega u z = - \frac{\omega^2 w}{2} = \text{constant},$$

by 307, and by 308, introducing the value $v^2 = 2gz$.

This vortex law when modified by deflection and friction becomes,

492. Case I.
$$v\omega = - \frac{\lambda}{k-c} \frac{c}{2} \omega^2 z = \text{const.}$$

Case II.
$$v\omega = - \frac{\lambda}{k} cz = \text{const.}$$

8. The inclination of the stream line to the isobars is,

491. Case I.
$$\cot i = + \frac{\lambda}{k-c} z.$$

Case II.
$$\cot i = + \frac{\lambda}{k} z.$$

9. The equation of continuity (163) is satisfied by the values in 490.

493. Case I.
$$\frac{\partial u}{\partial \omega} + \frac{u}{\omega} + \frac{\partial w}{\partial z} = - \frac{c}{2} - \frac{c}{2} + c = 0.$$

Case II.
$$\frac{\partial u}{\partial \omega} + \frac{u}{\omega} + \frac{\partial w}{\partial z} = + \frac{c}{\omega^2} - \frac{c}{\omega^2} + 0 = 0.$$

10. The equation for gradient has a term to express the unevaluated variation due to temperature effects, $f(t_x)$, and it becomes, for the radial component,

494.
$$- \frac{\mu}{\rho} G_r = - \frac{1}{\rho} \frac{\partial P}{\partial x}$$
$$= - \frac{c}{2} \omega \left[k-c + \lambda \frac{\lambda z}{k-c} + \frac{c}{2} \left(\frac{\lambda z}{k-c} \right)^2 \right] + f(t_x)$$
$$= - \frac{c}{2} \omega \left[k-c + \lambda \cot i + \frac{c}{2} \cot^2 i \right] + f(t_x).$$

11. The total velocity is

495.
$$q^2 = u^2 + v^2 + w^2 = \frac{u^2}{\sin^2 i} + w^2 = u^2 \left(\frac{1}{\sin^2 i} + \frac{4z^2}{\omega^2} \right) + f(t_x).$$

12. The variation of pressure can be expressed by

496.
$$\log P_o - \log P = \frac{(q^2 - q_o^2) + 2g(z - z_o)}{360862(1 + at)} + f_1(k) + f_2(i).$$

These formulæ are all collected in Table 121, page 602, of my International Cloud Report.

This system of formula applies directly only to the pure vortex motions that satisfy the assumed current function and velocity potential. The components u, v, w , are so simply

interrelated that it is usually possible to make enough observations of some sort from which to derive all the other vortex relations. Applications of them were made in the International Cloud Report to two cases; (1) The waterspout observed off Cottage City, Marthas Vineyard, Mass., August 19, 1896, on page 633; upon this important formation, a fuller report will be published. (2) The average velocities in a cyclone from the data in Table 126, as given on page 629 of the International Cloud Report. The outcome of these computations is to show that the natural stream lines of the atmosphere conform on the average to these formulæ. There are, however, wide divergences of such a type as to indicate that the pure vortex motion is seriously modified by several conflicting forces, and that the true problems for the meteorologist consist in discovering the nature and amount of these deviations of the currents of the atmosphere from the simple laws. This is in fact a task of great difficulty, but it has now become evident what should be the course of scientific development for meteorology. There is little use in a further discussion of the general theorems at the present time, but there is great need of procuring the right kind of observations for use in such problems. The Weather Bureau has accordingly been engaged in such a reconstruction of its data as will contribute to the solution of these problems for the United States. We have already published a large number of nephoscope velocities for the eastern half of the country; the velocities of the upper currents for the West Indies have been determined for about three years, July 1899-July 1902, and their computation will be commenced at once; similar nephoscope observations will be undertaken for the Rocky Mountain and Pacific districts, beginning about July 1902. Our barometric observations have been thoroughly reduced for the years 1873 to the present time, and the tables necessary for reductions to the three reference planes are in hand for the construction of daily maps at three levels, containing the system of isobars corresponding with them. It will be necessary to revise the temperature and vapor tension observations and reduce them to homogeneous systems before our data will be complete for the application of the theoretical equations to the observational data. It is desirable to put an end to general mathematical speculation in meteorology, and to substitute for it definite comparisons between observations and computations together with dependent solutions for the outstanding unknown quantities.

THE PROBLEMS OF THE AQUEOUS VAPOR CONTENTS OF THE ATMOSPHERE.

I shall allow myself only a few remarks regarding the methods which were used in my report for the discussion of the various complicated problems that concern the aqueous vapor contents of the atmosphere, because the details are too complex for a brief summary like this, and also because the work was given in such an extended form as to enable students to follow it without difficulty. There are, however, a few leading ideas to which attention may be especially directed, as they serve for an introduction to the subject in general.

There is collected in the International Cloud Report, Table 64, "Fundamental constants," a series of elementary constants in the English and metric systems, with the logarithms of the constants, and also a set of elementary formulæ which are most useful in meteorological studies. They cover nearly all the simple relations which constantly recur in manifold forms in the treatises and papers on meteorological subjects, and by transformation and combination a multitude of different relations can be readily obtained. Tables 63, 64, and 65 supply the basis for much descriptive matter commonly found in treatises, in so compact and accurate a form as to quite supersede the lengthy statements with which the same laws are usually presented, and this is a great convenience for the student and computer. Those who will take the trouble to become familiar with these tables will find much saving of

time in general work, and also they will be guarded from such errors of thought and statement as are likely to occur from not having these formulæ in mind, or accessible for convenient reference.

In treating the vapor problems I have referred all the formulæ to the ratio $\frac{e}{B}$, vapor tension divided by barometric pressure, as the most convenient and accurate argument for combination with another argument, as the height h , the temperature T , or the pressure B . The Table 67 summarizes the formulæ for the hypsometric reductions, and they are more fully explained in the forthcoming Barometry Report. The general

idea is that having found the ratio $\frac{e_o}{B_o}$ at the base of a column, the application of Hann's law for the diminution of the vapor pressure with the height gives the most accurate average law for computing the integral of the vapor tension throughout the entire column. A small secondary term can be added whenever our knowledge of the facts justifies such an increased degree of accuracy, though it is usually of little importance, especially for a series of observations where mean results are required.

In the development of the α , β , γ , δ stages of the adiabatic thermodynamic formulæ, the ratio $\frac{e}{B}$ is made the primary argument by the series of transformations given in Table 72. These formulæ are reduced to numerical tables, 94–102, and their accuracy is tested by comparing directly with the Hertzian logarithmic formulæ, as given in the examples of Table 108. Their use involves a series of solutions by trials, which though laborious, yet lead to perfectly rigorous results, and after a little practise it becomes quite easy to obtain the true trial values without much difficulty. The graphical diagrams of Hertz give only approximate values, because they throw out the vapor tension term in the critical places and thus render inaccurate the very problems they were designed to discuss. Special applications were made to finding the gradients of pressure, temperature, and vapor tension in the α , β , γ , δ stages, and the results are found in Tables 147 for metric measures, and in Tables 153 for English measures.

Finally the same tables were employed to discuss the important problem of the difference between an adiabatic atmosphere and the one given by the upper strata observations, whereby a new method was illustrated, with results in Table 162. The value of this computation depends, of course, upon the data B , T , e , adopted for the upper atmosphere, as measured by the balloon and kite ascensions. It was especially necessary to have the temperatures at high levels, and for this purpose I collected such material as was available up to the end of the year 1896, when I began this compilation, and for that purpose employed the 102 balloon ascensions enumerated in Table 155, embracing all those then available for the United States, England, France, Germany, and Russia. I expressed myself cautiously regarding the result, page 750, holding the computation as preliminary to a fuller one which would become possible when accurate observations had been accumulated for the upper air temperatures, and I have therefore had an interest in examining the Berlin report of the German balloon ascensions.⁴ In the first volume of this work are contained the data for each ascension, and in the *Meteorologische Zeitschrift*, October, 1901, page 449, H. Hergesell gives a summary of the resulting free air temperatures. I have extracted the observed temperatures from this report, interpolated them to each round 1,000-meter level, and computed the total temperature fall from the surface to the respective strata, with the result given in Table 21 and fig. 23. If the ascensions are divided into three sets, A , those reaching heights between the

surface and 5,000 meters, B , those between the surface and 10,000 meters, and C , those between the surface and 16,000

TABLE 21.—Comparison of several determinations of the total temperature change from the surface to high levels.

	A.	B.	C.	D.	E.	F.	G.	H.	I.
16000			-60.4	-68.3				-71.1	-115.0*
15000			-59.1	-66.0				-68.0	-106.0*
14000			-60.9	-62.5				-64.7	-97.0*
13000			-60.1	-63.5				-61.0	-88.0*
12000			-61.0	-60.3				-57.0	-79.0*
11000			-62.8	-52.7				-52.6	-70.0*
10000			-60.6	-48.5		-60	-62	-48.1	-61.0†
9000		-48.0	-57.0	-44.6	-56.8	-51	-56	-43.4	-54.5†
8000		-47.4	-51.0	-34.9	-48.7	-47	-48	-38.5	-47.9†
7000		-38.4	-44.8	-31.7	-39.8	-38	-41	-33.3	-39.6†
6000		-32.0	-37.5	-26.9	-34.6	-30	-34	-28.1	-32.9†
5000	-20.8	-25.5	-32.3	-23.1	-27.0	-25	-26	-22.8	-26.0†
4000	-15.0	-19.6	-28.0	-19.0	-20.7	-18	-21	-17.9	-19.9†
3000	-12.9	-14.3	-19.5	-13.0	-15.4	-13	-15	-13.1	-14.5†
2000	-7.9	-8.5	-15.8	-9.6	-9.9	-9	-8	-7.8	-9.0†
1000	-3.2	-3.7	-8.3	-3.8	-5.0	-4	-4	-3.9	-4.3†
0000	0.0	0.0	0.0	0.0	0.0	0	0	0.0	0.0†

A = 49 ascensions not above 5,000 meters in manned balloons.

B = 12 trips upward and 5 downward, not above 10,000 meters, in manned balloons.

C = 9 ascensions of unmanned balloons above 10,000 meters.

D = Bigelow's compiled data, Tables 156, I, II., International Cloud Report.

E = Berson's mean results, *Meteorologische Zeitschrift*, Oct. 1901, p. 449.

F = Teisserenc de Bort's mean results, *Meteorologische Zeitschrift*, Oct. 1901, p. 449.

G = Hergesell's mean results, *Meteorologische Zeitschrift*, Oct. 1901, p. 449.

H = Bigelow's mean results, Tables 157, I, II., International Cloud Report.

I = The mean of E , F , G up to 10,000 meters, and a gradient of 9° per 1,000 meters from 11,000 to 16,000 meters.

* Hergesell's assumed gradient 9° per 1,000 meters.

† Mean of E , F , G .

A and B , Berlin observations with manned balloons.
 C , Berlin observations with unmanned balloons.
 D , Bigelow's summary from all countries.
 E , Bigelow's adopted mean result.
 I , Berlin adopted mean result.

FIG. 23.—Total temperature fall from the surface to high levels several systems.

⁴ Wissenschaftliche Luftfahrten. Assmann und Berson. 3 Bänden. Berlin, 1899.

meters, we have the following remarkable data. Class *A* contains 49 ascensions of manned balloons, and gives a temperature fall of 20.8° at the 5,000-meter level; class *B* contains 12 upward and 5 downward trips of manned balloons and gives a fall of 25.5° at the 5,000-meter level, or 5° more than class *A*; class *C* contains 12 ascensions of unmanned balloons, with a fall of 32.3° at 5,000 meters, or 11.5° more than in class *A*, and 57° at 9,000 meters, or 9° more than in class *B*. This class shows also a fall of 60.6° at 10,000 meters and 60.4° at 16,000 meters. These widely different temperature falls by classes *A*, *B*, *C* may possibly be explained by those who are familiar with the circumstances, but the fact deserves attention; also the other fact that there is no temperature fall between 10,000 and 16,000 meters as observed in the Berlin unmanned balloon ascensions. In column *D* is given the result of my own compilation found by taking the mean of all the figures as they stand in Tables 156, I, II; and on fig. 21 the line *D* is seen to fall between *A* and *B* and to cross *C* at the height of 12,000 meters.

In his review of the Berlin ascensions H. Hergesell gave the Berson results as shown as in column *E*, the Teisserenc de Bort results as in column *F*, and his own results as in column *G*. He also stated the conclusion that above 10,000 meters the adiabatic rate of temperature fall in free air prevails, and this may be considered as 9.0° per 1,000 meters, as suggested by him. Column *I* is the mean value of *E*, *F*, *G*, up to 10,000 meters, and from that level to 16,000 the fall is calculated at 9.0° per 1,000 meters, these values being plotted on fig. 21. Finally, by taking the means of the data given in Tables 157, I, II, which was derived from Charts 78, 79, as constructed to determine the gradients for each month in the year, we have the data of column *H*, also plotted on fig. 21. It is seen that my adopted result, *H*, lies midway between *A* and *B*, and is a fair average of all the ascensions taken in the unmanned balloons, while the adopted Berlin result, *I*, is 45° lower at 16,000 meters, giving at that level a temperature of -115° approximately. There is a further consideration of importance to be noted in this connection. E. Rogovsky in his paper on the "Temperature and composition of the atmospheres of planets and the sun," Astrophysics, November, 1901, discusses the temperature of the interplanetary medium (according to Pouillet -142° C., Froelich -131° to -127°), and assumes

it to be -142° C. A fair supposition regarding the efficient depth of the atmosphere makes it 64,000 meters or about 40 miles, and hence we have the following data:

Height of atmosphere.	Bigelow.		Berlin.	
	Temperature.	Necessary gradients.	Temperature.	Necessary gradients.
Meters.	$^{\circ}$ C.	$^{\circ}$ C.	$^{\circ}$ C.	$^{\circ}$ C.
64,000	-142		-142	
16,000	-55	-1.8	-100	-0.9
Surface	15	-4.4	15	-7.2

If the temperature falls from 15° at the surface to -55° at 16,000 meters with a gradient of about -4.4° per 1,000 meters, then to reach -142° at 64,000 meters the gradient should on the average be -1.8° . It will be seen by my Charts 78 and 79, International Cloud Report, that I adopted an increasingly slower temperature fall with the height in the strata above 10,000 meters, in accordance with this general view. If the Berlin theory is assumed that a fall of 9.0° per 1,000 meters prevails above the 10,000-foot level, then it must somewhere rapidly decrease to a very small gradient in order not to diminish the interpolated temperatures far below that value assigned by certain astrophysicists to the celestial medium at the earth's distance from the sun. In fact the gradient becomes one-tenth of the adiabatic rate, which was actually assumed. If the temperature -260° C. is that of the interplanetary medium, as supposed by other writers, these inferences must be modified accordingly.

From these two considerations, (1) that my temperature system includes the data of the highest balloon ascensions, and (2) that my gradients are in harmony with the requirements of astrophysics, I shall let my computations on the heat difference between the adiabatic and the actual atmosphere stand as they were given in my report. The accurate measurement of the temperatures in the highest strata is a very difficult process, and all efforts to secure reliable results deserve the hearty support of meteorological physicists. There are several problems whose solution depends upon the possession of such data in a satisfactory form.

VII.—A CONTRIBUTION TO COSMICAL METEOROLOGY.¹

GENERAL REMARKS.

I have already published the results of certain computations and discussions on the subject of the direct connections between the variations of the solar output of energy, and the corresponding synchronisms in the meteorological elements of the earth's atmosphere. These are in particular, Solar and Terrestrial Magnetism, Weather Bureau Bulletin No. 21, 1898, and Eclipse Meteorology and Allied Problems, Weather Bureau Bulletin I, 1902, which include the substance of other minor papers related to this subject. The purpose of these studies has been, (1) to establish the fact that a synchronous connection does exist between the solar and the terrestrial forces, and (2) to derive the operation of these periodic movements so as to ultimately lead meteorology to a scientific understanding of the terrestrial seasonal climatic changes, and to a true basis for forecasts of weather conditions, at least one year in advance.

The difficulty of reaching a correct solution of this problem is well understood by those who have worked upon it, to reside in the unsteadiness of the solar output itself, and the numerous subordinate transformations of the energy, through the radiation, the general and local cyclonic circulations, till it culminates in a season having certain characteristics. The material for the study consists in the variations of the pressures, temperatures, and vapor tensions at many stations in different portions of the earth, in the fluctuations of the terrestrial magnetic field, in the changes of the spectrum energy of the solar and the aqueous vapor curves, and in the variations of the sun spots, the prominences, and the solar faculæ. The magnitude of the task involved in handling this material is such as to limit the attempt to deal with it to a few institutions having these subjects specially in charge. Among them the United States Weather Bureau has been able to make some contributions from time to time.

SUMMARY OF THE DISCUSSION OF 1898.

On pages 121-130, Bulletin No. 21, is given a brief account of an extensive discussion of the data then at hand, and the result was such as to show that there is a marked synchronism between the solar and terrestrial variations of energy. Fig. 24 serves to recall this fact and it shows that in the sun-spot period, 1878-1893, there was a true synchronism in the variation of the sun-spot areas, the European magnetic force, which is the resultant of the two components measured on a horizontal plane, and the American meteorological system. The latter includes a variation of temperature at 25 stations in the north-western portions of the United States, the pressure at 10 stations, the variable mean movements of the storms in latitude and longitude, and the movement of the tracks of the cold waves in latitude. Each of the two latter elements was derived from an exhaustive compilation of the coordinate positions of the cyclonic centers for the interval of fifteen years ending with 1893. It led me to the following summary:

The increase of solar magnetic intensity is synchronous with a diminution of temperature but with an increase of pressure, and this function persists throughout every phase of the research.

In spite of some irregularity, there is a distinct conformity in the general sweep of these curves, and also in the tendency to describe crests during the same years. Indeed, the occurrence of four subordinate crests in the 11-year period suggests strongly that a 2½-year period is superposed upon the long sweep of that periodic curve. Apparently this minor period is the basis of these seasonal variations of the weather conditions of the United States more than anything else, so that in long-range forecasting this period must be very carefully considered.

It was for the purpose of carrying this subject one step

further forward that the discussion of the data summarized in this present paper was undertaken. There has been considerable delay in completing the work on account of many other important duties.

It is evident that the terrestrial magnetic field affords the data which is most available for studying the fundamental periods in this solar-terrestrial synchronism. An exact quantitative computation for the several elements involves a very large amount of labor, and therefore it is important as an alternative to derive the periods by methods which shall give reliable proportional variations of the elements. The ideal treatment is to compute the total deflecting force of the magnetic field, by using the means of 24-hourly observations of the horizontal force, the declination, and the vertical force, taking out their daily component variations in rectangular coordinates ($dx, dy, dz,$) and combining them into polar coordinates s, α, β .

FIG. 24.

The next simpler method is to omit the vertical component, as one is tempted to do in consequence of the unreliable action of the Lloyd's balance, and turn the horizontal components, $dx, dy,$ into polar coordinates $\alpha, \beta,$ on the horizontal plane.

¹ Reprinted from the Monthly Weather Review for July, 1902.

Since it has been proved by computation that the east-west component, dy , derived from variations of the declination, practically disappears, as it should do by theory, we may adopt the variations of the horizontal component, dx , along the magnetic meridians, as the best single component for computation. This is all the more satisfactory because the bifilar horizontal magnet is the most efficient instrument in use in the magnetic observatories, and is generally free from objectionable features in its operations. I have, therefore, in this discussion, adopted the variations of the horizontal force, as shown by the 24-hourly means, on the ground that they are proportional to the total variations of the magnetic field and quite free from instrumental errors.

THE MAGNETIC OBSERVATIONS 1841-1899.

Accordingly, the magnetic horizontal force for the interval 1841-1899 has been submitted to a discussion, the result of which is summarized in this section. The synchronous action of the solar energy, as exhibited in the variation of the sun spots, the terrestrial aurora, the magnetic field, and several other phenomena, has been frequently developed, so that the general fact is admitted by all students, but it is now important to trace out this sympathetic movement in these cosmical forces more in detail, especially as they relate to the annual and seasonal variations in the earth's atmosphere. In the Proceedings of the Royal Society, volume 63, Mr. William Ellis, F. R. S., has exhibited this synchronism between Wolf's sun-spot numbers and the declination and horizontal force at the Greenwich Observatory for the interval 1841-1896. (Compare Bulletin I, 1902, page 105.) In his diagram not only do the curves present the same large sweeps, but also the minor variations appear simultaneously in the three curves. It is for the purpose of developing yet more distinctly these minor fluctuations that the compilation of the following magnetic observations was executed.

Instead of confining the study to a single observatory, it has been extended so as to practically include the entire earth, at least sufficiently to demonstrate that the variations are common to the whole terrestrial magnetic field. Thus, for different years we studied the records at the following stations:

- 1841-44. Toronto, St. Helena, Hobarton.²
- 1845. Greenwich, Toronto, Singapore, St. Helena, Cape of Good Hope, Hobarton.
- 1846-47. Toronto, St. Helena, Hobarton.
- 1848. Toronto, Greenwich, Hobarton.
- 1849-1870. Whatever was available, as Greenwich, Toronto, Madras, Batavia, Pavlosk, some of the data being unsatisfactory.
- 1871-77. Greenwich, Pavlosk.
- 1878-1885. Greenwich, Pavlosk, Vienna, Prague, Tiflis.
- 1886-1887. Los Angeles, Toronto, Greenwich, Paris, Pola, Prague, Pavlosk, Tiflis, Zi-ka-wei, Batavia.
- 1888. Greenwich, Prague, Pavlosk.
- 1889-90. Greenwich, Washington, Pavlosk.
- 1891. Greenwich, Prague, Pavlosk.
- 1892-99. Paris, Pola, Pavlosk.

By thus changing the stations it becomes impossible that the peculiar action of any set of instruments, should such exist, can impose a bias upon the final result. It seems to me that it makes no difference what three stations are chosen to represent the cosmical variation of the magnetic field, as indicated by the horizontal force which is proportional to the total force. Three stations are desirable in order to eliminate the local impulses of the field, and if they had been available I should have used the same three stations throughout, all reduced to the C. G. S. system of units, for the sake of having rigorous quantitative results. The data of this paper limit it to showing relative synchronisms, but these are quite sufficient for our purposes in the present stage of meteorology.

The horizontal force, as given by the means of twenty-four successive hourly ordinates, or by a smaller number of selected hours in some cases, was considered, and the *daily variation from the normal horizontal force* was computed either numerically or graphically. (Compare the methods of Bulletins Nos. 2 and 21.) In some of the years the normal force was found by drawing a mean line through the monthly trace of the curve, as plotted from the daily means; in other years the daily variation was computed from the numerical data contained in the published reports. In all years from 1841-99 the horizontal trace was graphically transferred to curves and distributed in the period of 26.68 days, whose epoch is June 13.72, 1887; the exact adopted period was 26.67928 days, as given in the January Ephemeris. (See Bulletin No. 21, page 120.) Therefore throughout this interval the several curves, generally three in number, are plotted on the sheets, so that the irregularities as well as the agreements are open to inspection. On examining this long series of curves in succession, it is evident that a decided change occurs in the amplitudes of the variable curve with regard to the normal base line which was superposed upon each of them. In some years the curves are flat and quiet, so that the ordinates are small, and the curves are free from violent or spasmodic impulses. In other years, on the contrary, the curves sway about roughly and are much disturbed, great irregularities being superposed upon them. My procedure was to measure, at least proportionally, the area which is inclosed between the variable curve and the average base line and to integrate these areas in the 26.68-day period, the 11-year period, and throughout the interval 1841-99. This was accomplished by measuring these ordinates from day to day, taking the mean of those stations selected for the given day, usually three in number, and transferring these mean ordinates with their plus and minus signs to the Ephemeris Tables. For by taking the ordinates from day to day, that is to say, at frequent intervals along the curve, their *sum is proportional to the area developed between the bounding curve and the base line*. If the actual area had been read off by means of a planimeter, the relative variation between different epochs would have been the same. These relative numbers were transferred to tables, showing the direct type curves in black ink and the inverse type curves in red ink, which can not be reproduced in this connection. These figures were now summed together by the year, the direct type and the inverse type separately, but no account is given here of this summation. Also the figures were summed across each period, without regard to their signs, so that the total divergence of amplitude might appear as a sort of integral. In each year there are thirteen or fourteen periods, but when the successive years are placed in the 11-year period, and five or six of these in the final summary, then the mean dates of the total ephemeris can be restored by simply applying 26.68 days to the mean of the January dates, that is to January 15.44, as on page 106, of Bulletin No. 21.

Table 22, "Total variations of the horizontal force for the earth generally, arranged in 26.68-day periods," contains an exhibit of these integral sums of the included areas for each 26-day period within the limits of the computation; the sums are also given for each year for five successive 11-year cycles, and for the general mean; and, finally, the differences between the annual sums for each successive year and also the average annual variation. In taking the mean of each bottom row of periods, the mean of the numbers visible, usually 7 or 8, was increased proportionally to 11, in order to make it comparable with the other 13 periods; the action of the ephemeris causes the periodic skip in the 14th period. There was one other place in which an arbitrary change was introduced, namely, wherever the disturbance in the negative direction, at a few of the largest perturbations, exceeded -0.00040 C. G. S. units, the disturbance ordinate was computed at this value. There are only a few excessive disturbances of the horizontal force

² Now called Hobart Town, or Hobart, Tasmania.

TABLE 22.—Total variations of the horizontal magnetic force for the earth generally, arranged in 26.58-day periods.

11-YEAR PERIOD, 1841-1851.

26-day period.	1841.	1842.	1843.	1844.	1845.	1846.	1847.	1848.	1849.	1850.	1851.	Means.
1.....	171	142	130	155	177	101	193	282	152	204	141	168
2.....	170	181	159	153	139	100	200	272	160	225	112	169
3.....	204	168	130	151	111	238	289	277	170	137	104	180
4.....	254	165	126	154	102	231	220	223	362	147	153	193
5.....	226	209	178	135	122	228	196	161	170	200	164	181
6.....	134	175	109	96	142	153	137	199	114	261	155	153
7.....	154	200	75	132	82	110	155	105	130	317	140	149
8.....	169	93	138	118	79	127	130	216	141	217	162	145
9.....	143	89	119	113	85	155	164	196	129	178	279	150
10.....	145	160	96	182	99	220	296	120	161	80	250	164
11.....	219	136	84	242	108	208	231	85	188	160	187	168
12.....	197	176	58	147	97	139	257	152	121	183	159	153
13.....	211	240	68	142	62	131	320	106	182	111	191	160
14.....	239	107	130		126	95		76	206	184		160
Sums.....	2,516	2,301	1,600	1,900	1,631	2,239	2,788	2,470	2,389	2,604	2,197	2,293
Differences.....	-215	-701	+300	-269	+608	+549	-318	-81	+215	-407		333

11-YEAR PERIOD, 1852-1862.

26-day period.	1852.	1853.	1854.	1855.	1856.	1857.	1858.	1859.	1860.	1861.	1862.	Means.
1.....	225	118	175	86	183	105	281	162	174	281	231	183
2.....	248	138	134	124	134	117	186	239	224	169	182	172
3.....	146	138	179	126	157	125	241	150	269	208	199	176
4.....	258	132	170	146	142	171	262	206	273	207	194	196
5.....	189	168	116	107	151	188	324	225	161	131	144	173
6.....	153	162	100	81	128	90	190	139	168	233	171	147
7.....	128	139	126	90	95	125	177	225	272	202	194	161
8.....	151	118	91	141	111	114	163	150	305	176	208	155
9.....	107	163	125	112	163	192	134	271	230	212	169	171
10.....	154	189	127	100	165	143	194	221	168	267	253	180
11.....	137	121	79	131	121	213	199	379	115	187	281	179
12.....	152	161	104	106	92	231	205	168	148	289	166	166
13.....	167	148	72	93	124	282	196	330	256	250	187	192
14.....	129	117		151	168		174	151		240	170	179
Sums.....	2,315	2,015	1,558	1,594	1,937	2,096	2,926	3,016	2,763	3,061	2,749	2,430
Differences.....	+118	-300	-457	+36	+343	+159	+830	+90	-253	+298	-312	291

11-YEAR PERIOD, 1863-1873.

26-day period.	1863.	1864.	1865.	1866.	1867.	1868.	1869.	1870.	1871.	1872.	1873.	Means.
1.....	163	120	89	135	85	80	183	166	120	149	137	130
2.....	140	99	121	195	80	117	82	204	196	232	181	150
3.....	159	117	94	96	80	159	91	184	165	167	118	130
4.....	128	102	85	105	93	129	143	201	161	210	144	136
5.....	116	103	78	68	115	139	217	272	137	175	126	141
6.....	113	121	147	84	73	111	150	209	222	202	139	143
7.....	99	105	98	149	125	94	111	173	151	227	95	130
8.....	122	151	156	140	67	90	129	250	144	196	101	141
9.....	133	138	113	135	69	90	159	230	160	215	125	142
10.....	131	133	89	159	118	159	150	290	141	193	107	151
11.....	115	176	126	124	58	178	99	319	111	225	130	151
12.....	123	146	122	66	65	128	131	263	163	228	142	143
13.....	66	93	69	98	71	101	189	251	109	144	120	118
14.....		95	171	98		98	281		186	199		177
Sums.....	1,608	1,699	1,558	1,643	1,099	1,673	2,123	3,012	2,157	2,762	1,665	1,983
Differences.....	-1,141	+91	-141	+85	-541	+574	+450	+889	-855	+605	-1,097	616

11-YEAR PERIOD, 1874-1884.

26-day period.	1874.	1875.	1876.	1877.	1878.	1879.	1880.	1881.	1882.	1883.	1884.	Means.
1.....	120	108	114	112	172	160	134	302	224	230	235	174
2.....	239	212	146	82	146	97	191	251	165	241	223	181
3.....	199	125	100	95	151	127	135	171	144	283	196	158
4.....	202	132	83	64	141	120	171	182	421	223	233	179
5.....	152	114	93	108	152	162	215	145	215	210	251	163
6.....	159	128	57	107	191	187	173	235	264	192	203	172
7.....	106	97	85	148	168	161	160	220	256	224	296	175
8.....	159	82	73	113	135	177	310	187	295	197	184	174
9.....	115	75	79	118	103	156	250	157	183	356	255	168
10.....	112	139	92	90	141	204	221	264	344	194	242	186
11.....	184	100	105	92	116	159	278	149	281	229	218	174
12.....	132	105	110	98	187	182	176	238	486	287	264	206
13.....	115	91	120	128	176	194	219	219	290	186	267	182
14.....	97	79		70	180	172		160	141		244	157
Sums.....	2,091	1,587	1,266	1,425	2,165	2,257	2,636	2,880	3,809	3,052	3,311	2,449
Differences.....	+426	-504	-321	+159	+740	+92	+379	+244	+929	-757	+259	437

TABLE NO. 22.—Continued.

11-YEAR PERIOD, 1885-1895.

26-day period.	1885.	1886.	1887.	1888.	1889.	1890.	1891.	1892.	1893.	1894.	1895.	Means.
1.....	265	136	161	177	128	101	158	261	279	143	182	181
2.....	223	224	162	169	153	116	174	286	200	389	156	204
3.....	194	276	136	222	143	126	164	298	220	205	186	197
4.....	197	225	172	218	147	102	182	409	207	254	191	210
5.....	324	198	163	223	131	94	287	278	209	210	200	211
6.....	183	156	130	161	130	102	139	301	223	189	156	170
7.....	273	174	172	140	137	115	140	330	230	147	181	185
8.....	141	207	178	149	138	123	157	245	302	250	232	193
9.....	177	263	190	157	189	115	178	211	212	295	191	198
10.....	247	272	208	171	184	131	302	174	229	269	170	214
11.....	171	242	179	184	225	164	162	255	312	128	247	206
12.....	201	209	149	205	251	140	191	179	191	295	256	209
13.....	221	127	137	139	136	117	214	231	247	176	164	179
14.....	257		194	127		110	178	150		155	216	188
Sums.....	3,074	2,709	2,391	2,533	2,092	1,656	2,626	3,608	3,061	3,135	2,728	2,745
Differences.....	-237	-365	-318	+142	-441	-436	+970	+982	-547	+74	-407	447

1896-1899.

26-day period.	1896.	1897.	1898.	1899.	Means.	Average for the whole interval, 1841-1899.
1.....	152	165	152	171	161	166
2.....	220	169	179	151	180	176
3.....	177	169	260	249	214	176
4.....	254	188	178	190	203	186
5.....	273	203	153	157	197	178
6.....	194	195	190	166	186	162
7.....	205	203	121	134	191	165
8.....	185	144	128	160	142	158
9.....	209	100	208	103	155	164
10.....	229	115	169	132	161	176
11.....	227	124	190	145	172	175
12.....	218	113	148	109	155	172
13.....	261	178	166	102	177	168
14.....		218	127		173	172
Sums.....	2,834	2,285	2,369	1,920	2,467	
Differences.....	+106	-549	+84	-449		

below this limit, and I did not wish to distort the average annual numbers with these great abnormalities.

The revised Wolf's table of the sun-spot numbers, by Prof. A. Wolfer, Meteorologische Zeitschrift, May 1902, page 197,³ has been used to give the curve of the sun-spot variations, it being unimportant for this discussion whether the observed or smoothed numbers are employed.

The result of this computation, "Variation of the sun-spot numbers and the amplitude area numbers," is shown in fig. 25, the figures of Table 22 being transferred thereto. The annual sums were plotted so as to give the horizontal force curve, and the mean sums for the successive 11-year periods were plotted for the mean curve. This curve brings out three variations with extraordinary clearness: (1) The 35-year period, with maxima in 1855 and 1890, a minimum at about 1868, another probable minimum at about 1833, and one more at about 1903. After this exhibit there can be little doubt of the existence of this long period variation, discussed by Lockyer and others, and it is certain that a continuation of this method of computation will eventually fix the characteristics of this period with exactness. The fall from maximum to minimum seems to occupy thirteen years, and the rise from minimum to maximum requires a longer time, probably twenty-two years. (2) The 11-year period is seen to be in exact synchronism throughout the interval 1841-1899 with the sun spots and the horizontal force taking the curve as a whole, but there are superposed upon it a series of abrupt minor variations, which, as stated above, it is chiefly desirable to obtain for comparison with our meteorological

³This table had been originally communicated to the Monthly Weather Review and the proof sheets sent to Professor Wolfer for revision, so that as published in the Monthly Weather Review, April, 1902, page 175, the figures have the full authority of Professor Wolfer, and it is believed no typographical error exists therein.—ED.

FIG. 25.—Variation of the sun-spot numbers and the amplitude area numbers.

logical data. (3) These subordinate crests of energy indicate that in the rise and fall of the 11-year period there is a series of spasmodic impulses, generally one in ascending the curve and two in descending, which, added to the maximum crest itself, makes four minor crests to be superposed upon the mean 11-year curve, as mentioned in the opening paragraphs, and shown in fig. 24. In the ascending branch the successive annual changes are not equal to the mean value, and this

FIG. 26.—Semiannual period in the horizontal force of the terrestrial magnetic field, arranged for six successive 11-year periods.

FIG. 27.

branch must evidently be considered as produced by a secondary system of crests, even though the 11-year line is not deeply

indented. The discussion of this 2 $\frac{1}{2}$ -year period will be resumed in a later section of this paper.

If the mean values of the fourteen periods as collected in the 11-year periods and indicated in Table 22 be plotted successively, the result is as shown in fig. 26. We find that there is a distinct semiannual period in the horizontal force areas, with maxima at March 22 and September 22, and minima at June 22 and December 22, thus indicating that it depends upon the orbital relations of the earth to the sun. But, furthermore, it is noted that the same 35-year period is indicated within this semiannual period, since there is a distinct minimum in the period 1874-1884, and maxima in the 1852-1862 and 1896-1900 periods, as measured by their amplitudes. Also, there is, apparently, a tendency for the spring maximum to surpass the autumn maximum, whenever they are strongest within the 35-year period. We have here indicated a field of research of importance in mechanical astronomy, since it implies that another force besides simple Newtonian gravitation is binding the sun and the earth together. It becomes an interesting problem to discover whether these magnetic forces are capable of fulfilling the outstanding theoretical requirements involved in the orbital perturbations of the earth and the other planets. It becomes, also, a further argument, in addition to those presented in my bulletins, Solar and Terrestrial Magnetism, and Eclipse Meteorology and Allied Problems, to show that the sun is a great magnetised sphere, in whose external field the earth is immersed. On fig. 27 I have copied Chart No. 19, page 106, of Bulletin No. 21, which shows the curve of the frequency of the direct type in the 26.68-day period. Its crests regularly precede those in the semiannual orbital period by a small interval, and there must be a physical reason for this divergence, such as explained in my other papers.

COMPARISON OF THE VARIATIONS OF THE SOLAR PROMINENCES WITH THOSE OF THE TERRESTRIAL HORIZONTAL MAGNETIC FORCE FOR THE INTERVAL 1874-1900.

It is well understood that the variations of the sun-spot frequency constitute only one of the manifestations of the changes in the output of the solar energy; the frequency of the hydrogen prominences, or of the faculae, and of the extensions of the solar corona are other forms of the display of this variable force. Indeed, there is reason to believe that the sun spots are a somewhat sluggish type of the variable impulses, although the first to be studied, on account of the ease with which the spots are observed. Since scientific processes of observation have improved of late years, it has become possible to measure the frequency of the prominences and of the faculae with precision, so that a continuous record is now being made of these types of solar energy. The prominences have been observed by Tacchini since 1873 and the faculae by Hale and others for several years, so that it is now possible to add to the sun-spot record that of each of these two phenomena. The prominences are distributed all over the surface of the sun, and the relative frequency has been determined in 10-degree zones between latitudes $\pm 90^\circ$ annually since 1873, so that we possess a prominence curve of relative frequency extending through more than two 11-year cycles. Through the courtesy of Sir Norman Lockyer, of the Solar Physics Observatory, South Kensington, London, I have had an opportunity to see some advance copies of different sets of curves of a very valuable character prepared by him for a paper published by the Royal Society, in which this subject is discussed. It is gratifying to note that his work confirms my curves of 1898 and is in agreement with those presented in this paper. I reproduce the Lockyer-Tacchini prominence curve, which represents the mean frequency in all latitudes for the years 1874-1900. It is found at the head of fig. 28. It shows a large curvature synchronous with the sun-spot frequency in the 11-year cycle, and also a series of minor crests of a characteristic nature. Underneath

this curve is placed the series of minor variations which were found in the horizontal magnetic force, as shown in fig. 25, after the 11-year cycle curve has been eliminated. The re-

markable synchronism between these curves can not escape recognition, except after the year 1894, when an extra minor crest is developed in the horizontal force. If these two curves are compared with the 15-year systems exhibited on fig. 24, it is evident that my paper of 1898 had detected the same synchronism, not only throughout the curve of sun-spot frequency, but also throughout the whole European magnetic field and the entire American meteorological system.

THE VARIATIONS OF ATMOSPHERIC PRESSURE OVER THE ENTIRE EARTH.

In the course of my studies into this set of phenomena, including the solar and terrestrial magnetic fields and the meteorological elements, it became evident that in cosmical problems we should be compelled to deal with the variations of small quantities in meteorology, such as a few hundredths of an inch of pressure and a few degrees of temperature. It was, therefore, necessary to carefully exclude all possible sources of error due to the imperfect methods of observation and reduction, in order that variations arising from such causes might not be falsely attributed to cosmical forces. The result of such a discussion of the barometric pressures for the United States will be found in Report of the Chief of the United States Weather Bureau for 1901, Volume II. A similar rediscussion of the temperature and the vapor tension will be executed as soon as practicable. It is important that comparable rigorously homogeneous systems should be prepared by other weather services which possess continuous long series of records of the meteorological elements. Pending the preparation of such revised systems, I have collected together a considerable number of sets or series of barometric pressures, taken in different parts of the earth, and have reduced them to a homogeneous basis, as well as I could, from a study of the published data. There are still annoying discontinuities at many stations, due to changes in the elevations of the barometers. It is also probable that the instrumental errors and the methods of reduction employed still need to be thoroughly examined.

Table 23, "The variations of the annual atmospheric pressure in many districts of the earth," serves to indicate, at least approximately, the relations of the annual barometric pressure variations to the changes in the solar output. It contains a summary of the results in the several countries where long series of barometric observations exist. It is arranged in an order which will bring out a remarkable feature of the pressure variations, as will be briefly indicated. The table gives the mean data for comparatively large districts; it is divided into groups and the mean pressures for these groups are transferred to fig. 28, which has been already mentioned. The following catalogue shows the stations that were employed in the discussion:

- Northeast China.*—Zi-ka-wei, Peking, Vladivostok.
- Japan.*—Tokio, Nagasaki, Hieroshima, Osaka, Kioto.
- North India.*—Leh, Murree, Simla.
- Central India.*—Darjeeling, Lahore, Lucknow, Calcutta.
- South India.*—Pachmari, Bangalore, Nagpur, Bombay, Madras.
- Batavia and Mauritius.*
- North New South Wales.*—Albury, Bathurst, Deniliquin.
- South New South Wales.*—Goulburn, Newcastle, Sidney.
- Kimberley.*—Kimberley, Bloemfontein.
- Inland Cape Colony.*—Grahamtown, Lovedale, Aliwal North.
- Coast Cape Colony.*—Cape Town, Port Elizabeth, East London, Mossel Bay.
- North Russia.*—Archangel, St. Petersburg.
- East Russia.*—Moscow, Katharinenburg.
- Russia and Southwestern Siberia.*—Odessa, Tiflis, Baku.
- Central Siberia.*—Tomsk, Barnaul, Irkutsk.
- France.*—Paris.
- Spain.*—Madrid, Lisbon, San Fernando, Coimbra.
- South Europe.*—Pola, Budapest, Kalocsa, O'Gyalla, Vienna.
- United States.*—Pacific coast States, 20 stations; northern

FIG. 28.—Comparison of the solar prominence variations with those of the terrestrial horizontal magnetic force and the atmospheric pressures over the entire earth.

TABLE NO. 23.—The variations of the annual mean atmospheric pressures in many districts of the earth, in units of 0.001 inch.

Stations.	1873.	1874.	1875.	1876.	1877.	1878.	1879.	1880.	1881.	1882.	1883.	1884.	1885.	1886.	1887.	1888.	1889.	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	
Northeast China	- 8	- 8	+ 4	-16	+12	+32	-20	-32	- 4	+24	+24	+12	+24	+20	- 4	+ 4	+24	- 8	+ 4	- 4	+ 4	+12	-12	- 4	+24	-12	-12	
Japan	0	+ 8	+ 4	0	+ 4	+12	-12	+ 4	- 8	+12	+ 8	+ 4	+ 8	+16	- 8	-12	0	-24	- 8	- 8	+ 8	+16	- 8	-12	+16	- 8	+ 8	
North India			+ 6	- 1	+ 7	+18	- 9	- 3	+11	-13	-16	+ 1	+ 3	+ 5	- 3	+ 7	+13	- 9	+20	+ 6	+14	-10	+13	+18	+ 7	- 1	+14	
Central India			- 8	- 2	+35	+16	- 6	0	+ 3	- 9	- 4	+12	+16	+ 7	+ 3	+ 8	+ 9	-11	+16	+24	+ 6	- 8	+ 3	+ 5	- 7	-16	- 3	
South India			-10	- 8	-31	+ 6	-14	0	+ 7	- 9	- 5	+11	+20	- 1	+ 2	+19	+ 8	- 3	+17	- 9	- 6	-10	+ 2	+ 5	- 7	-15	+12	
Batavia	- 4	+ 4	- 8	+ 8	-43	+ 4	-20	+ 8	+16	0	-16	-24	-20	- 8	- 8	-20	+12	+ 4	+24	-12	- 4	- 4	0	+20	+ 8	-12	+12	
Mauritius	0	- 6	- 3	+ 8	+ 7	+ 3	+16	+24	+22	+16	- 7	-12	0	+ 8	+12	+12	+17	+ 2	+14	-16	+10	+22	-13	+14	0	-13	+12	
Means	- 3	0	- 2	- 2	+20	+11	- 9	0	+ 9	+ 3	- 2	+ 1	+ 7	+ 5	- 2	+ 3	+12	- 7	+15	- 4	+ 1	- 3	- 1	+10	+ 7	-11	- 2	
North New South Wales	+17	+ 9	-32	- 1	+55	-13	-39	-21	+46	- 3	-12	- 4	+12	+15	0	+42	+26	- 1	+54	- 6	-36	- 9	+ 7	+ 7	+ 5	-21		
South New South Wales	+ 7	- 2	-38	- 7	+40	-35	-37	- 4	+31	-18	- 5	+ 5	+43	+11	- 2	+44	+30	- 7	+27	-23	-47	- 3	- 3	- 2	+ 1	-13		
Means	+12	+ 4	-35	- 4	+50	-24	-38	-13	+39	-11	- 9	+ 1	+28	+13	- 1	+43	+28	- 4	+41	-15	-42	- 6	+ 2	+ 3	+ 3	-17		
Kimberly				+ 5	- 2	+ 1	-25	+ 2	+16	+10	-14	- 4	+11	+38	+12	+34	+ 7	-23	-14	-15	- 3	- 5	+ 3	+12	- 4	- 9	+ 4	
Inland Cape Colony				+14	+ 9	+ 5	-16	- 7	+25	-17	- 5	+ 5	- 8	+11	+29	+ 4	+27	- 5	+ 3	-11	-11	+ 2	- 3	+ 3	-11	- 2	+19	
Coast Cape Colony				+30	+ 8	+ 4	- 3	+17	+18	+ 4	-26	+25	+17	+11	+14	+18	+ 2	-15	+ 5	-24	-13	- 2	+ 3	+15	+ 5	+ 2	+18	
Means				+16	+ 5	+ 3	-15	+ 4	+20	+10	-15	+ 9	+ 7	+20	+18	+ 7	+12	-14	- 2	-17	- 9	- 2	+ 1	+10	- 3	- 3	+14	
North Russia			-24	+55	+39	+35	-75	+20	-71	+16	+24	+43	+55	+ 4	+55	-35	- 8	+79	+47	+20	- 8	-47	-16	+24	+59	+63	+20	-43
East Russia			-20	-12	+ 8	+63	-16	-12	-67	- 4	-32	+32	-12	-20	-24	-47	-55	+51	+20	0	+36	-16	- 8	-12	+36	+39	+16	-24
Southwest Siberia			+28	-20	+ 8	-32	-24	-12	- 8	+ 8	+16	+20	+20	+ 4	+16	+ 8	-16	+ 8	+28	+36	-12	-16	+12	-24	+12	+4	+24	-20
Central Siberia			0	-12	-24	-67	+40	-32	-16	- 4	-12	+ 8	0	+ 8	-28	-32	-24	+43	-39	-20	+43	+16	-20	+20	- 8	+24	+4	+24
Means			- 3	+ 3	+ 8	+49	-19	- 9	-41	+ 4	- 1	+26	+16	- 1	-31	-27	-26	+45	+14	+ 9	+15	-16	- 8	+ 2	+25	+33	+14	-16
Paris	0	+39	+24	-43	-28	-24	-12	+20	0	+ 8	+20	+35	+12	-43	+39	0	+12	+16	+24	-32	+20	+16	-43	+51	+20	+32		
Spain	-12	+20	+12	-28	+16	0	-12	+16	-32	+55	+32	+ 4	-32	-20	-28	- 8	-20	-16	+12	-39	- 4	-20	-47	+35	+24	0	- 8	
South Europe	+12	+12	+ 4	-24	-12	-35	-28	+32	+16	-32	+12	+39	-12	-16	-20	+12	-12	+16	+35	-16	+12	+28	-43	+16	+16	+12	+16	
Means	0	+24	+13	-32	- 8	-20	-17	+23	- 5	+32	-21	+26	-11	-26	+10	+ 1	+ 7	+ 5	+25	-29	+ 9	+21	-44	+34	+20	+15	+ 4	
Pacific coast	+15	+ 3	+22	0	-12	-31	- 4	+19	- 1	+ 7	+16	-27	-15	- 2	+ 1	-16	-15	+16	+ 3	+ 6	+ 9	+ 8	+ 8	- 4	+ 7	- 1	+ 1	
North Plateau				-20	- 8	-28	-24	- 7	- 4	- 4	+18	- 7	- 5	0	- 6	+11	+16	+14	- 7	+ 8	-18	- 5	+10	+ 5	+12	+12	- 4	
South Plateau				+ 3	- 9	-25	- 9	- 8	-13	- 1	+11	- 5	+ 8	+ 5	+ 1	+ 4	+22	+20	- 5	+ 2	-12	+14	+12	+ 8	+6	+1	- 3	
Lake region	-15	+28	0	-12	+11	-51	+10	- 6	+ 2	+ 7	+15	+ 2	-20	0	+ 1	+24	+ 8	+12	+ 7	+20	-32	-12	+10	+ 6	+11	-11	+ 2	
West Gulf States	- 1	+19	+ 8	+ 7	+ 2	-49	+ 7	+13	-15	+ 1	+15	- 6	-12	- 9	- 5	+ 8	+ 6	+11	+ 1	+ 4	-21	+12	+ 6	+ 9	0	- 3	0	
North Atlantic States	-10	+34	+ 4	-20	+ 5	-67	+10	+21	- 8	-20	+31	- 1	-33	- 6	- 1	- 1	-11	- 1	+15	- 8	-16	-17	- 8	+10	+ 8	- 3	+11	
South Atlantic States	- 1	+31	+ 9	- 2	-10	-58	+17	+25	-14	-13	-18	- 8	-31	-13	- 2	+ 4	-12	+12	+ 6	+ 6	+19	+ 9	- 7	+15	+ 2	- 1	+ 4	
Means	- 2	+23	+ 9	- 6	- 3	-44	+ 1	+ 8	- 8	+ 6	+18	- 7	-15	- 4	- 2	+ 5	+ 2	+12	+ 3	+ 5	-16	+ 6	+ 2	+ 7	+ 7	- 1	+ 2	
North Argentina	- 4	+12	+ 8	+ 8	-20	+ 4	+ 8	- 4	-12	+20	+16	+16	+16	+16	- 8	-20	+ 8	+20	- 8	+ 8	+24	0	- 4					
Central Argentina	-16	+20	+20	+12	-12	-16	+16	-12	-20	+ 4	+ 4	0	- 4	+20	+12	- 8	+12	+28	- 4	+ 8	+24	+16	- 8	-12	+ 8	- 8		
Means	-10	+16	+14	+10	-16	+10	+12	- 8	-16	+12	+10	+ 8	+ 6	+18	+ 2	-14	+10	+24	- 6	+ 8	+24	+ 8	- 6	-12	+ 8	- 8		

Plateau, 33 stations; southern Plateau, 19 stations; Lake region, 31 stations; west Gulf States, 41 stations; North Atlantic States; 26 stations; South Atlantic States, 32 stations; total number of United States stations, 202.

North Argentina.—Villa Formosa, Corrientes, Salta, Tucuman, Santiago, Goya, Hernandarias.

Central Argentina.—Cordoba, San Juan, Parana, Rosario, Carcaraña, Estancia San Juan, Buenos Ayres, Chacra de Matanzas, Bahia Blanca, Colonia Chabut.

In all cases the mean annual pressures were extracted from the observatory reports; these were reduced to the same elevation of the barometer, usually that for 1899, and all known corrections were applied. The mean for the homogeneous series was computed and then the variation of each year from this mean, the result being always changed, if necessary, into units of 0.001 inch in the English system. These annual variations were plotted as curves on sheets for the several countries, so that the several districts could be studied for their characteristic types. The stations were finally grouped as indicated in the catalogue, and the larger district means, including about all the region having the same type of curve, were transferred to fig. 28. It was very interesting to study these local curves, and to note that the same pressure variations in fact prevail over very large districts of the earth, though varying from one region to another. The variations were also transferred to charts of the earth, one for each year, and it was found that while there is an irregularity from year to year, it was possible to discover some very suggestive features. I regret that these charts can not be reproduced in this connection. Some years show that in North America and South America the annual pressure prevails in excess, or that the variation is positive, as 1874, 1875, 1883, 1890, 1892, 1897. Others show that the entire Northern Hemisphere is in defect as a whole, as 1876, 1878, 1879, 1885, 1887, 1893. Others show the Northern Hemisphere to be in excess, as 1883, 1896, 1897. Other years are more irregular. I have the impression that there is a westward movement of the defect in pressure, or of the negative residuals; and that there are similar groups separated by intervals of seven or eight years. This subject will require an

exhaustive study by meteorologists in the future, and much valuable information will be extracted from it.

If the positive values of the pressure variations be added together for each year, and also the negative values by themselves, the result may be indicated as it is plotted in the curves of fig. 29. The upper curve is for the positive and the lower for the negative summation, but these curves show, since they rise and fall together, that these values do not cancel each other. The curves match fairly well with the prominence curve, and I take it to mean that *some external force is at work to raise and lower the total atmospheric pressure by a small amount from year to year.* It is probable that a more rigorous discussion would eliminate certain distortions of this curve, and show that it synchronizes very closely with the curve of the variations of solar energy. If this proves to be so, it raises some exceedingly interesting questions in cosmical meteorology.

FIG. 29.—Positive and negative pressure variations over the earth as a whole for successive years, on a scale of relative numbers.

It is interesting to compare the results of this series of annual variations, 1873–1899, with those of the series, 1874–1884, studied by H. H. Hildebrandsson,⁴ the latter, however, extending the details to the monthly values. The data of the Barome-

⁴Quelques recherches sur les centres d'action de l'atmosphère, par H. H. Hildebrandsson, Stockholm, 1897.

try Report make it possible to do this readily for the United States with little additional labor.

Returning to fig. 28, if we compare the successive pressure groups with the prominence curve, it will be seen that India and southeastern Asia are in very close synchronous agreement. This synchronism extends also to New South Wales, the Indian Ocean, and even to south Africa. In Siberia and Russia the synchronism begins to break a little and seems to be transferred somewhat toward the right, although this may be due in part to defective data. In Europe and in the United States, while the same curve is developed as to the number of the maxima and minima, the synchronism becomes more irregular. In South America, on the other hand, the synchronism is resumed very distinctly, but the *entire curve is reversed as referred to India and the Eastern Hemisphere*. Thus we perceive that around the Indian Ocean the synchronism is clearly developed; it weakens in Europe and North America, and it becomes a distinct reversal in South America. I presume that this remarkable phenomenon is due to the fact that the Pacific-Indian Ocean is quite free from frequent cyclonic disturbances, as is also South America, and that the atmospheric pressure surges back and forth between these two central or southern hemispheres, or else slowly rotates about the entire earth, probably from east to west. In North America and

Europe, while the type curve reappears less perfectly, it still exists, and the disturbance may be due to the turbulent cyclonic circulation, which prevails over this region of the earth in marked contrast with the quiescent circulation of the other regions. It is, however, of much importance to have shown that changes in the annual atmospheric pressure of the earth synchronize approximately with the typical output of solar energy.

From this rapid survey of the cosmical meteorological problem, it is obvious that meteorology has large interests in solar and terrestrial magnetism. The annual reports of magnetic observatories are usually published several years after the records are made, hence, if meteorology is to insure any progress in seasonal forecasting, it evidently must possess its own magnetic apparatus, so that the state of the solar-terrestrial field may be known in connection with current meteorological phenomena. It must be conceded that considerable scientific skill will be required to bring this system of cosmical forces into control for the benefit of mankind, but I do not see how it can be doubted that the true pathway of research is already open before us. It is to be hoped that meteorologists generally will take up these cosmical problems, and compute the necessary homogeneous systems, so that it may become possible to advance promptly to practical results.

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